Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 1

Exercise 1

Given the function

$f(x,y) = 3x^3 + y^3$ defined over the domain $D \equiv 9x^2 + 3y^2 \leq 48$, compute its absolute maxima and minima.

1) The value of the minimum is ****.5****

2) The value of the minimum is ****.9****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.3****

5) The value of the minimum is ****.6****

Exercise 2

Compute $\int_D (5y)\,dx\,dy$ for $D = \{0 \leq 3x - 9y \leq 9,\ 0 \leq 3x + 4y \leq 7\}$

1) -2.0213

2) -0.421302

3) -0.621302

4) -0.121302

5) -1.9213

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (2,1)$.

1) $H(2,1) = -4.85559$

2) $H(2,1) = 3.79204$

3) $H(2,1) = 3.07185$

4) $H(2,1) = 0.5$

5) $H(2,1) = -5.11991$

Exercise 4

Consider the vector field $F(x,y,z) = (-x, y, z)$ and the parameterized surface

$X: [-1,1] \times [0,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1 + uv, 2v, 2u)$

Compute $\int_X F.$

1) 34.4 2) 0.8 3) 8. 4) 20.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 2

Exercise 1

Given the function

$f(x,y) = -3x^3 + 4y^3$ defined over the domain $D \equiv 9x^2 + 30y^2 \leq 786$, compute its absolute maxima and minima.

1) The value of the maximum is ****.3****

2) The value of the maximum is ****.5****

3) The value of the maximum is ****.4****

4) The value of the maximum is ****.7****

5) The value of the maximum is ****.8****

Exercise 2

Compute $\int_D (2x + 2y)\,dx\,dy$ for $D = \{0 \leq -3x - y \leq 7,\ 0 \leq -5y \leq 4\}$

1) -5.35111

2) -4.45111

3) -7.05111

4) -5.85111

5) -4.25111

Exercise 3

Compute the mean curvature for $X(u,v) = \{2u, 1, v\}$ at the point $(u,v) = (9, 10)$.

1) $H(9, 10) = -7.56929$

2) $H(9, 10) = -6.99326$

3) $H(9, 10) = 0$

4) $H(9, 10) = -2.91388$

5) $H(9, 10) = -7.99699$

Exercise 4

Consider the vector field $F(x,y,z) = (0, -z, y)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (v^2, 2uv, 3u^2)$

Compute $\int_X F$.

1) -5.5 2) -25.3 3) -4.3 4) -15.1

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 3

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x\,y - y^2}{x^2 + y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -2

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -44

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 225,\ z \ge 8\sqrt{x^2 + y^2}\right\}$

1) -16.3753

2) 32.7507

3) -16.3753

4) 54.5845

5) 16.3753

Exercise 3

Compute the center of curvature for $C(t) = \left\{-2\,t^2, -t, -3\,t^2\right\}$ at the point $t = 3$.

1) $\{-54.0769, 1404., -81.1154\}$

2) $\{-615.677, 1263.6, 620.885\}$

3) $\{-334.877, 1684.8, -923.515\}$

4) $\{1069.12, 2246.4, -642.715\}$

5) $\{1069.12, 982.8, 1182.48\}$

Exercise 4

Consider the vector field $F(x,y,z) = (\,x\,, -y\,, -z\,)$ and the parameterized curve

$\sigma : [\,-1\,, 0\,] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (\,1 + t\,, 2\,, 2 + t^2\,)$

Compute $\int_\sigma F$.

1) 4.2 2) -1.5 3) $3.$ 4) 6.3

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 4

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3y^3}{(x^2+y^2)^{3/2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -43

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 37

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 121,\ z\ge 15\sqrt{x^2+y^2}\right\}$

1) 9.87869

2) 2.46967

3) 6.17418

4) 0.617418

5) 3.70451

Exercise 3

Compute the center of curvature for $C(t)=\left\{-2t^2,\ -2t^2,\ 1+2t\right\}$ at the point $t=-1$.

1) {7.1, −1.4, 3.4}

2) {−6.5, −6.5, 17.}

3) {−1.4, −16.7, 11.9}

4) {−1.4, −21.8, 30.6}

5) {−9.9, −8.2, 1.7}

Exercise 4

Consider the vector field $F(x,y,z)=(-y,0,0)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(4t,1,1+t^2)$

Compute $\int_\sigma F$.

1) −4. 2) 7.6 3) −14.8 4) 4.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 5

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^4 - y^4}{-9x - 6x^2 + x^4 - 3x^5 - 6x^6 + 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 106

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −78

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \le 196,\ z \ge 3(x^2 + y^2)\}$

1) 141.979

2) 263.675

3) 101.413

4) 233.251

5) −91.272

Exercise 3

Compute the center of curvature for $C(t) = \{t, -2t^2, 3t^2\}$ at the point $t = 2$.

1) {−624., 267.123, −130.285}

2) {−707.2, −107.277, −88.6846}

3) {−624., 183.923, 77.7154}

4) {−416., −24.0769, 36.1154}

5) {−374.4, 59.1231, 160.915}

Exercise 4

Consider the vector field $F(x,y,z) = (0, 0, -z)$ and the parameterized curve

$\sigma: [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (3t^2, 2t^2, 4)$

Compute $\int_\sigma F$.

1) 0.6 2) 1.4 3) 0.5 4) 0.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 6

Exercise 1

Given the function

$f(x,y) = -4x^3 - 5y^3$ defined over the domain $D \equiv 6x^2 + 30y^2 \leq 486$, compute its absolute maxima and minima.

1) The value of the maximum is ****.5****

2) The value of the maximum is ****.0****

3) The value of the maximum is ****.1****

4) The value of the maximum is ****.8****

5) The value of the maximum is ****.6****

Exercise 2

Compute $\int_D (x + 3y)\,dx\,dy$ for $D = \{0 \leq -3x - 8y \leq 3,\ 0 \leq -x - 4y \leq 6\}$

1) -6.5625

2) -3.7625

3) -5.2625

4) -6.9625

5) -5.0625

Exercise 3

Compute the mean curvature for $X(u,v) = \{2u, 2u^2, v\}$ at the point $(u,v) = (7, 1)$.

1) $H(7, 1) = 6.7088$

2) $H(7, 1) = -3.57657$

3) $H(7, 1) = -6.88303$

4) $H(7, 1) = 0.00018083$

5) $H(7, 1) = -4.98422$

Exercise 4

Consider the vector field $F(x,y,z) = (x, 0, -z)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2 + uv, uv, 2v^2)$

Compute $\int_X F.$

1) 6.76667 2) 13.9667 3) 8.86667 4) 3.16667

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 7

Exercise 1

Given the function

$f(x,y) = -2x^3 - y^3$ defined over the domain $D \equiv 15x^2 + 9y^2 \leq 699$, compute its absolute maxima and minima.

1) The value of the minimum is ****.3****

2) The value of the minimum is ****.0****

3) The value of the minimum is ****.5****

4) The value of the minimum is ****.4****

5) The value of the minimum is ****.1****

Exercise 2

Compute $\int_D (2x + 3y)\,dx\,dy$ for $D = \{0 \leq -5x + 3y \leq 6,\ 0 \leq -x + 6y \leq 4\}$

1) -0.398765

2) -0.0987654

3) -1.59877

4) 0.201235

5) -1.49877

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (1, 3)$.

1) $H(1, 3) = 1.54722$

2) $H(1, 3) = -3.98993$

3) $H(1, 3) = -2.32566$

4) $H(1, 3) = 0.5$

5) $H(1, 3) = 4.40141$

Exercise 4

Consider the vector field $F(x,y,z) = (0, y, z)$ and the parameterized surface

$X: [-1, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v, 2 + uv, v + 2uv)$

Compute $\int_X F.$

1) -40.3333 2) -17.3333 3) 5.66667 4) -9.33333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 8

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2x^4+2y^4}{2x+x^2+x^4-2x^5+x^6-y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 30

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −57

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 100,\ z\ge 11\left(x^2+y^2\right)\right\}$

1) 0.

2) 9.95068

3) −9.95068

4) 25.5875

5) 14.2153

Exercise 3

Compute the center of curvature for $C(t)=\left\{-2t^2,\ -t^2,\ 3+t\right\}$ at the point $t=2$.

1) {−87., −43.5, −47.1}

2) {−24.2, −12.1, −157.}

3) {−102.7, −74.9, −78.5}

4) {22.9, 97.8, −266.9}

5) {−134.1, −153.4, −219.8}

Exercise 4

Consider the vector field $F(x,y,z)=(z,y,0)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(1+t,t,2t)$

Compute $\int_\sigma F$.

1) 0. 2) −3. 3) 2.5 4) −2.2

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 9

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^3+2x^2y}{x^2+y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 4

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 43

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 144,\ z\ge 11\sqrt{x^2+y^2}\right\}$

1) 35.6711

2) 22.2944

3) 25.267

4) −14.863

5) 14.863

Exercise 3

Compute the center of curvature for $C(t)=\left\{2t^2,\ 3t^2,\ 3-2t\right\}$ at the point $t=3$.

1) {−509.692, 574.962, 493.5}

2) {−298.192, −482.538, 1198.5}

3) {−86.6923, −553.038, 211.5}

4) {265.808, 151.962, 282.}

5) {54.3077, 81.4615, 705.}

Exercise 4

Consider the vector field $F(x,y,z)=(0,y,-z)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(3t,t+2t^2,1+2t)$

Compute $\int_\sigma F$.

1) −3.1 2) −0.5 3) −0.6 4) 1.7

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 10

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{2y}{\sqrt{x^2+y^2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 22

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −35

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 196,\ z\ge 9\sqrt{x^2+y^2}\right\}$

1) 35.1503

2) 31.6353

3) −14.0601

4) 7.03006

5) 45.6954

Exercise 3

Compute the center of curvature for $C(t)=\left\{-3t^2,\ -3t^2,\ -2+t\right\}$ at the point $t=-1$.

1) {11.9167, −30.0833, 133.}

2) {32.9167, 11.9167, 21.}

3) {−23.0833, −23.0833, 91.}

4) {4.91667, 4.91667, 126.}

5) {−9.08333, −9.08333, 70.}

Exercise 4

Consider the vector field $F(x,y,z)=(-y,z,0)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(1+t^2,t^2,3)$

Compute $\int_\sigma F.$

1) −2.5 2) −11.5 3) −7.6 4) 5.9

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 11

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{-2x-x^2+x^3-x^4-x^5+y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -28

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 9

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 169,\ z\geq 8\left(x^2+y^2\right)\right\}$

1) 33.024

2) 42.9313

3) 75.9553

4) 89.1649

5) 36.3265

Exercise 3

Compute the center of curvature for $C(t)=\left\{t^2,-t^2,2t\right\}$ at the point $t=-3$.

1) {17.2, 15.2, 75.6}

2) {28., −28., 108.}

3) {38.8, 58.4, 75.6}

4) {−69.2, −60.4, 183.6}

5) {82., −17.2, 75.6}

Exercise 4

Consider the vector field $F(x,y,z)=(0,z,-x)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2+2t^2,1+t,1+t^2)$

Compute $\int_\sigma F$.

1) 2.66667 2) 7.26667 3) 9.46667 4) 2.06667

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 12

Exercise 1

Given the function

$f(x,y) = 5x^3 + y^3$ defined over the domain $D \equiv 30x^2 + 3y^2 \leq 492$, compute its absolute maxima and minima.

1) The value of the minimum is ****.2****

2) The value of the minimum is ****.1****

3) The value of the minimum is ****.5****

4) The value of the minimum is ****.6****

5) The value of the minimum is ****.4****

Exercise 2

Compute $\int_D (4y)\,dx\,dy$ for $D = \{0 \leq 7x - 5y \leq 2,\ 0 \leq -3x + 4y \leq 1\}$

1) 1.80769

2) −0.392308

3) 0.507692

4) −0.192308

5) 0.307692

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (1, 9)$.

1) $H(1, 9) = -3.43777$

2) $H(1, 9) = 6.32546$

3) $H(1, 9) = -7.94821$

4) $H(1, 9) = 0.5$

5) $H(1, 9) = 3.88453$

Exercise 4

Consider the vector field $F(x,y,z) = (z, x, 0)$ and the parameterized surface

$X: [0, 1] \times [-1, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (v, 2v, u)$

Compute $\int_X F.$

1) −3.6 2) −2. 3) 3.2 4) −5.8

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 13

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^2+2xy}{x^2+y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 6

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 31

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 121,\ z\ge 3\sqrt{x^2+y^2}\right\}$

1) 143.052

2) 414.852

3) 128.747

4) −28.6105

5) 42.9157

Exercise 3

Compute the center of curvature for $C(t)=\left\{2t^2,\ -3+3t,\ -3t^2\right\}$ at the point $t=-3$.

1) {194.192, 651., −500.538}

2) {54.6923, 465., −82.0385}

3) {8.19231, 372., −314.538}

4) {−270.808, 186., −500.538}

5) {287.192, 186., −314.538}

Exercise 4

Consider the vector field $F(x,y,z)=(-z,0,y)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2+t,t^2,2t)$

Compute $\int_\sigma F$.

1) 2.56667 2) −0.933333 3) 1.66667 4) 2.26667

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 14

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 12

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −46

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 1,\ z\ge 9\sqrt{x^2+y^2}\right\}$

1) −1.98719

2) 0.0128099

3) 1.11281

4) 0.71281

5) 0.81281

Exercise 3

Compute the center of curvature for $C(t)=\left\{1+t,\ 3t^2,\ -3t^2\right\}$ at the point $t=1$.

1) {−71., 9.08333, −9.08333}

2) {−56.8, 30.3833, −23.2833}

3) {−134.9, −54.8167, 26.4167}

4) {−78.1, 72.9833, 19.3167}

5) {−56.8, 65.8833, −72.9833}

Exercise 4

Consider the vector field $F(x,y,z)=(z,0,0)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(1,2+2t,t+2t^2)$

Compute $\int_\sigma F$.

1) 2. 2) 1.1 3) −3.2 4) 0.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 15

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^4 + 2x^3 y - 3y^4}{(x^2+y^2)^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -16

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -39

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 4,\ z \ge 8\sqrt{x^2+y^2}\right\}$

1) 1.52939

2) 1.92939

3) −1.07061

4) 0.129385

5) −0.870615

Exercise 3

Compute the center of curvature for $C(t) = \{2t^2, -3t^2, 1-2t\}$ at the point $t = 2$.

1) {−121.992, 26.2385, 229.9}

2) {−121.992, 26.2385, 250.8}

3) {−80.1923, 68.0385, 355.3}

4) {24.3077, −36.4615, 209.}

5) {212.408, 109.838, 376.2}

Exercise 4

Consider the vector field $F(x,y,z) = (0, -x, y)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2, 2t, t^2)$

Compute $\int_\sigma F$.

1) −9.26667 2) −4.76667 3) −2.66667 4) −8.06667

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 16

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3+2y^3}{4x+4x^2+x^3-2x^4-4x^5-2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 18

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 40

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 16,\ z\geq 15\left(x^2+y^2\right)\right\}$

1) -0.830815

2) 3.48942

3) 1.66163

4) 2.82477

5) 1.16314

Exercise 3

Compute the center of curvature for $C(t)=\left\{-2t^2,\ 2t^2,\ 2+2t\right\}$ at the point $t=2$.

1) {-24.5, 24.5, -126.}

2) {-49.7, -88.9, -138.6}

3) {-37.1, 74.9, -88.2}

4) {88.9, 11.9, -25.2}

5) {-100.1, 49.7, -163.8}

Exercise 4

Consider the vector field $F(x,y,z)=(-z,0,y)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2t,2t,4t)$

Compute $\int_\sigma F$.

1) -3.7 2) 3. 3) 0. 4) -3.3

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 17

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^3 - 3y^3}{-9x - 3x^2 + x^3 + 3x^4 + 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 145

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −78

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \le 64,\ z \ge 2(x^2 + y^2)\}$

1) -7.10543×10^{-15}

2) 48.727

3) 131.563

4) −9.74541

5) 73.0905

Exercise 3

Compute the center of curvature for $C(t) = \{-3t^2, -3t^2, -1 - t\}$ at the point $t = -2$.

1) {310.117, −151.483, −57.7}

2) {−36.0833, −36.0833, −577.}

3) {−555.383, −382.283, −807.8}

4) {194.717, 310.117, −1096.3}

5) {−497.683, −151.483, −923.2}

Exercise 4

Consider the vector field $F(x,y,z) = (0, -z, x)$ and the parameterized curve

$\sigma: [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (t^2, t^2, 1 + 2t)$

Compute $\int_\sigma F$.

1) −1.33333 2) −8.53333 3) −5.93333 4) 0.666667

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 18

Exercise 1

Given the function

$f(x,y) = 5x^3 + 3y^3$ defined over the domain $D \equiv 45x^2 + 18y^2 \leq 1908$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.6****

3) The value of the maximum is ****.2****

4) The value of the maximum is ****.4****

5) The value of the maximum is ****.0****

Exercise 2

Compute $\int_D (2x + y)\,dx\,dy$ for $D = \{0 \leq -4x - 2y \leq 8,\ 0 \leq 2x + 2y \leq 2\}$

1) −7.8

2) −9.8

3) −6.8

4) −8.

5) −6.7

Exercise 3

Compute the mean curvature for $X(u,v) = \{3u, u^2, v\}$ at the point $(u,v) = (10, 10)$.

1) $H(10, 10) = -4.11992$

2) $H(10, 10) = -7.79543$

3) $H(10, 10) = 0.794973$

4) $H(10, 10) = -5.82778$

5) $H(10, 10) = 0.000362691$

Exercise 4

Consider the vector field $F(x,y,z) = (0, z, 0)$ and the parameterized surface

$X: [-1, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2uv, 4uv, 4u)$

Compute $\int_X F$.

1) 69.6333 2) −29.0667 3) 105.333 4) 21.3333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 19

Exercise 1

Given the function

$f(x,y) = 5x^3 - 5y^3$ defined over the domain $D \equiv 15x^2 + 30y^2 \leq 540$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.1****

3) The value of the maximum is ****.0****

4) The value of the maximum is ****.2****

5) The value of the maximum is ****.4****

Exercise 2

Compute $\int_D (2x + 3y)\,dxdy$ for $D = \{0 \leq 9x - 8y \leq 1,\ 0 \leq -8x + 3y \leq 9\}$

1) -1.07071

2) -3.37071

3) 0.429291

4) -3.37071

5) -1.37071

Exercise 3

Compute the mean curvature for $X(u,v) = \{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v) = (1, 4)$.

1) $H(1, 4) = 0.915543$

2) $H(1, 4) = 2.54794$

3) $H(1, 4) = 5.90868$

4) $H(1, 4) = 0.0883883$

5) $H(1, 4) = -7.52317$

Exercise 4

Consider the vector field $F(x,y,z) = (-x, -y, 0)$ and the parameterized surface

$X: [0, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1, 1 + v^2, 2u + v)$

Compute $\int_X F.$

1) -0.2 2) -2. 3) -7. 4) -1.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 20

Exercise 1

Given the function

$f(x,y) = -5x^3 - 2y^3$ defined over the domain $D \equiv 30x^2 + 12y^2 \leq 672$, compute its absolute maxima and minima.

1) The value of the maximum is ****.1****

2) The value of the maximum is ****.9****

3) The value of the maximum is ****.3****

4) The value of the maximum is ****.4****

5) The value of the maximum is ****.2****

Exercise 2

Compute $\int_D (4y)\,dx\,dy$ for $D = \{0 \leq 5x + 2y \leq 9,\ 0 \leq -2x - 2y \leq 3\}$

1) -48.7

2) -49.9

3) -49.9

4) -49.8

5) -49.5

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (4, 6)$.

1) $H(4, 6) = -4.75774$

2) $H(4, 6) = -0.067376$

3) $H(4, 6) = 0.5$

4) $H(4, 6) = 4.62201$

5) $H(4, 6) = 7.35208$

Exercise 4

Consider the vector field $F(x,y,z) = (x, 0, 0)$ and the parameterized surface

$X: [-1, 0] \times [-1, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (u + 2v^2, 2u + 2uv, 2uv)$

Compute $\int_X F.$

1) -3.8 2) 1.6 3) 1.5 4) 0.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 21

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2x^3+3y^3}{-2x-2x^2+x^3+2x^4+y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 22

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −2

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 36,\ z\ge 10\left(x^2+y^2\right)\right\}$

1) −1.6824

2) 5.608

3) −3.9256

4) 11.7768

5) 15.1416

Exercise 3

Compute the center of curvature for $C(t)=\left\{2t^2,\ -t^2,\ -1+3t\right\}$ at the point $t=-3$.

1) {163.2, 7.9, 107.4}

2) {109.5, −81.6, 196.9}

3) {55.8, −27.9, 179.}

4) {199., 7.9, 214.8}

5) {163.2, 133.2, 53.7}

Exercise 4

Consider the vector field $F(x,y,z)=(0,x,y)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2t^2,2+2t^2,t)$

Compute $\int_\sigma F$.

1) 18.8333 2) 24.3333 3) 5.33333 4) 7.33333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 22

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^3 - 3x^2y}{x^2+y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -40

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -39

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2+y^2+z^2 \le 4,\ z \ge 7\sqrt{x^2+y^2}\right\}$

1) 1.7684

2) 0.168398

3) 0.768398

4) -0.431602

5) 0.568398

Exercise 3

Compute the center of curvature for $C(t) = \left\{2t^2, -1-t, 2t^2\right\}$ at the point $t = -3$.

1) {486.625, -1124.5, -291.875}

2) {-464.875, -1211., -637.875}

3) {486.625, -173., -32.375}

4) {54.125, -865., 54.125}

5) {313.625, -1557., -378.375}

Exercise 4

Consider the vector field $F(x,y,z) = (0, 0, x)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2, 1, 1+2t)$

Compute $\int_\sigma F$.

1) 3.6 2) 16. 3) 6.8 4) 4.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 23

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{-4x - 4x^2 + x^4 - 2x^5 + 2x^6 + 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -2

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -15

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 121,\ z \geq 8\ (x^2 + y^2)\}$

1) 37.7981

2) 66.1467

3) −23.6238

4) 23.6238

5) 49.61

Exercise 3

Compute the center of curvature for $C(t) = \{2 - 2t,\ t^2,\ -2t^2\}$ at the point $t = 2$.

1) {139.4, 61.6, −33.}

2) {16.4, 37., −49.4}

3) {82., 12.4, −24.8}

4) {106.6, 37., 16.2}

5) {155.8, 86.2, 49.}

Exercise 4

Consider the vector field $F(x,y,z) = (0, 0, -y)$ and the parameterized curve

$\sigma: [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2 + 2t, 3t^2, 2t^2)$

Compute $\int_\sigma F$.

1) −0.4 2) 0.4 3) −1.1 4) 0.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 24

Exercise 1

Given the function

$f(x,y) = 3x^3 - 4y^3$ defined over the domain $D \equiv 27x^2 + 6y^2 \leq 978$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.5****

3) The value of the maximum is ****.9****

4) The value of the maximum is ****.3****

5) The value of the maximum is ****.1****

Exercise 2

Compute $\int_D (2x + 2y)\,dx\,dy$ for $D = \{0 \leq -5x - 3y \leq 7,\ 0 \leq 4y \leq 9\}$

1) -1.175

2) -3.475

3) -1.575

4) 0.325

5) 0.025

Exercise 3

Compute the mean curvature for $X(u,v) = \{u, 3u, v\}$ at the point $(u,v) = (4, 7)$.

1) $H(4, 7) = 7.3226$

2) $H(4, 7) = -3.94829$

3) $H(4, 7) = -3.36604$

4) $H(4, 7) = 0$

5) $H(4, 7) = -5.19955$

Exercise 4

Consider the vector field $F(x,y,z) = (0, z, x)$ and the parameterized surface

$X: [0, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v^2, 2u + v^2, v + 2uv)$

Compute $\int_X F.$

1) 0. 2) 3.2 3) 1.1 4) 2.8

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 25

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^3 - 2y^3}{-4x - 4x^2 + x^3 - 4x^4 - 4x^5 + 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -13

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -36

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 64,\ z \ge 8\left(x^2 + y^2\right)\right\}$

1) 12.4687

2) 36.1592

3) 19.9499

4) −6.23435

5) 13.7156

Exercise 3

Compute the center of curvature for $C(t) = \left\{-t^2, -t^2, -3t\right\}$ at the point $t = -3$.

1) {−86.85, −72.45, −93.6}

2) {−14.85, 21.15, −136.8}

3) {−43.65, −86.85, −21.6}

4) {−65.25, −7.65, −115.2}

5) {−29.25, −29.25, −72.}

Exercise 4

Consider the vector field $F(x,y,z) = (0, y, 0)$ and the parameterized curve

$\sigma : [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2t^2, 2 + 2t^2, 2t + 2t^2)$

Compute $\int_\sigma F$.

1) 0. 2) −1.2 3) −0.5 4) 0.7

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 26

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^4+2y^4}{-9x-3x^2+x^4-3x^5+3x^6+3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 159

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −168

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 1,\ z\ge 7\left(x^2+y^2\right)\right\}$

1) 0.209114

2) 0.509114

3) 0.509114

4) −1.79089

5) 1.90911

Exercise 3

Compute the center of curvature for $C(t)=\left\{2t^2,\ -2+2t,\ -t^2\right\}$ at the point $t=-3$.

1) {269.2, 428.8, 79.8}

2) {54.8, 268., −27.4}

3) {−79.2, 187.6, 160.2}

4) {162., 134., −0.6}

5) {215.6, 80.4, 79.8}

Exercise 4

Consider the vector field $F(x,y,z)=(y,x,0)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(1,2t,2+2t^2)$

Compute $\int_\sigma F$.

1) 16.4 2) 8.8 3) 4. 4) −3.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 27

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^2+2xy}{\sqrt{x^2+y^2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 22

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 14

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 81,\ z\ge 2\sqrt{x^2+y^2}\right\}$

1) 161.19

2) 32.238

3) -48.357

4) 225.666

5) 241.785

Exercise 3

Compute the center of curvature for $C(t)=\left\{-3t^2,\ -3+t,\ t^2\right\}$ at the point $t=3$.

1) {568.65, -1516.2, 1001.75}

2) {568.65, -1516.2, -297.85}

3) {27.15, -433.2, 1001.75}

4) {-730.95, -1841.1, 893.45}

5) {-81.15, -1083., 27.05}

Exercise 4

Consider the vector field $F(x,y,z)=(0,-z,x)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2,1+t^2,2t)$

Compute $\int_\sigma F$.

1) 2.66667 2) -4.73333 3) 3.66667 4) 1.26667

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 28

Exercise 1

Given the function

$f(x,y) = -4x^3 - 2y^3$ defined over the domain $D \equiv 24x^2 + 15y^2 \leq 759$, compute its absolute maxima and minima.

1) The value of the maximum is ****.4****

2) The value of the maximum is ****.8****

3) The value of the maximum is ****.3****

4) The value of the maximum is ****.0****

5) The value of the maximum is ****.5****

Exercise 2

Compute $\int_D (3x)\,dx\,dy$ for $D = \{0 \leq 4x + 5y \leq 6,\ 0 \leq -4x + 5y \leq 1\}$

1) -1.85938

2) -0.459375

3) 1.14063

4) 0.140625

5) 0.340625

Exercise 3

Compute the mean curvature for $X(u,v) = \{2u, 2, v\}$ at the point $(u,v) = (8, 2)$.

1) $H(8, 2) = -5.64169$

2) $H(8, 2) = 5.28785$

3) $H(8, 2) = 0$

4) $H(8, 2) = 0.611803$

5) $H(8, 2) = -1.0142$

Exercise 4

Consider the vector field $F(x,y,z) = (0, z, y)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2u^2 + v, 2uv + v^2, 2 + v)$

Compute $\int_X F.$

1) -3.5 2) -0.5 3) 0.8 4) 0.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 29

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3 - y^3}{-3x - 2x^2 + x^3 - 2x^4 + y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -25

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 13

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 9,\ z \ge 14\left(x^2 + y^2\right)\right\}$

1) -0.402129

2) 0.997871

3) 1.99787

4) 1.09787

5) 2.89787

Exercise 3

Compute the center of curvature for $C(t) = \left\{t^2, 3 - 2t, -3t^2\right\}$ at the point $t = -1$.

1) $\{-7., -15.3, -4.5\}$

2) $\{-7., -27.2, -19.8\}$

3) $\{-5.3, -6.8, 2.3\}$

4) $\{3.2, -17., -9.6\}$

5) $\{-7., -11.9, -16.4\}$

Exercise 4

Consider the vector field $F(x,y,z) = (x, 0, z)$ and the parameterized curve

$\sigma : [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (t, 3t^2, 2t^2)$

Compute $\int_\sigma F$.

1) 1.7 2) $-3.$ 3) 1.1 4) $0.$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 30

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2x^2y^2 - 3xy^3}{(x^2+y^2)^{3/2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -2

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -22

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 16,\ z\geq 9\sqrt{x^2+y^2}\right\}$

1) 0.819832

2) 1.81983

3) -0.480168

4) 0.119832

5) -0.280168

Exercise 3

Compute the center of curvature for $C(t)=\{-3t^2,\ 3t^2,\ t\}$ at the point $t=-2$.

1) {-151.283, -367.117, 921.6}

2) {-554.483, -136.717, 691.2}

3) {79.1167, 266.483, 921.6}

4) {194.317, -309.517, 230.4}

5) {-36.0833, 36.0833, 576.}

Exercise 4

Consider the vector field $F(x,y,z)=(z,0,0)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2t,2+t,2)$

Compute $\int_\sigma F$.

1) -12. 2) 1.2 3) 4. 4) 18.4

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 31

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^4+2y^4}{-3x-9x^2+x^4-6x^5+3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 1

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −10

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 81,\ z\ge 15\left(x^2+y^2\right)\right\}$

1) −7.60587

2) 8.45096

3) 16.9019

4) 21.9725

5) 19.4372

Exercise 3

Compute the center of curvature for $C(t)=\left\{-3t^2,\ 2t^2,\ 1+3t\right\}$ at the point $t=-2$.

1) {−37.0385, 24.6923, 139.667}

2) {60.2615, −3.10769, 56.2667}

3) {74.1615, −100.408, 84.0667}

4) {−106.538, −17.0077, 56.2667}

5) {46.3615, −44.8077, 125.767}

Exercise 4

Consider the vector field $F(x,y,z)=(z,y,x)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2+2t,t^2,3)$

Compute $\int_\sigma F$.

1) −13.2 2) 12. 3) 34.8 4) 16.8

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 32

Exercise 1

Given the function

$f(x,y) = 4x^3 - 5y^3$ defined over the domain $D \equiv 30x^2 + 45y^2 \leq 2370$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.7****

3) The value of the minimum is ****.1****

4) The value of the minimum is ****.6****

5) The value of the minimum is ****.3****

Exercise 2

Compute $\int_D (x+y)\,dx\,dy$ for $D = \{0 \leq -4x - 9y \leq 4,\ 0 \leq -4x - 3y \leq 4\}$

1) -1.63333

2) -1.73333

3) 0.866667

4) -0.333333

5) -0.433333

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (3, 10)$.

1) $H(3, 10) = -8.02886$

2) $H(3, 10) = 0.5$

3) $H(3, 10) = 3.92266$

4) $H(3, 10) = -6.65157$

5) $H(3, 10) = 4.96242$

Exercise 4

Consider the vector field $F(x,y,z) = (x, z, 0)$ and the parameterized surface

$X: [-1, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2uv, uv, 2v^2)$

Compute $\int_X F.$

1) 4.8 2) -6.4 3) -12. 4) -0.8

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 33

Exercise 1

Given the function

$f(x,y) = -2x^3 + 3y^3$ defined over the domain $D \equiv 6x^2 + 18y^2 \leq 312$, compute its absolute maxima and minima.

1) The value of the maximum is ****.7****

2) The value of the maximum is ****.3****

3) The value of the maximum is ****.8****

4) The value of the maximum is ****.6****

5) The value of the maximum is ****.9****

Exercise 2

Compute $\int_D (5y)\,dx\,dy$ for $D = \{0 \leq 5x - y \leq 4,\ 0 \leq 6x + 2y \leq 6\}$

1) 2.25

2) -0.84375

3) 1.40625

4) -0.84375

5) 1.54688

Exercise 3

Compute the mean curvature for $X(u,v) = \{3u, u^2, v\}$ at the point $(u,v) = (5, 6)$.

1) $H(5, 6) = 8.46518$

2) $H(5, 6) = -2.54725$

3) $H(5, 6) = -5.66133$

4) $H(5, 6) = -2.1254$

5) $H(5, 6) = 0.00263622$

Exercise 4

Consider the vector field $F(x,y,z) = (-z, 0, y)$ and the parameterized surface

$X: [-1, 0] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (v, 2v^2, 2u)$

Compute $\int_X F.$

1) 2.8 2) 8.8 3) -16. 4) -4.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 34

Exercise 1

Given the function

$f(x,y) = x^3 - y^3$ defined over the domain $D \equiv 6x^2 + 3y^2 \leq 108$, compute its absolute maxima and minima.

1) The value of the minimum is ****.5****

2) The value of the minimum is ****.4****

3) The value of the minimum is ****.0****

4) The value of the minimum is ****.7****

5) The value of the minimum is ****.3****

Exercise 2

Compute $\int_D (2y)\,dx\,dy$ for $D = \{0 \leq -9x + 9y \leq 9,\ 0 \leq -4x - 7y \leq 7\}$

1) 1.32645

2) -1.77355

3) -0.173554

4) -1.97355

5) 1.22645

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \text{Cos}[u],\ v^2 \text{Sin}[u],\ v\}$ at the point $(u,v) = (1, 2)$.

1) $H(1, 2) = -0.707901$

2) $H(1, 2) = 6.48177$

3) $H(1, 2) = -8.4576$

4) $H(1, 2) = 0.0160502$

5) $H(1, 2) = 3.99978$

Exercise 4

Consider the vector field $F(x,y,z) = (-x, 0, z)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2u + 2v, 2, 2uv)$

Compute $\int_X F.$

1) 3.3 2) 0.8 3) 3.9 4) 0.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 35

Exercise 1

Given the function

$f(x,y) = 5x^3 + 2y^3$ defined over the domain $D \equiv 45x^2 + 3y^2 \leq 1623$, compute its absolute maxima and minima.

1) The value of the maximum is ****.6****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.5****

4) The value of the maximum is ****.3****

5) The value of the maximum is ****.4****

Exercise 2

Compute $\int_D (3x + 2y)\,dx\,dy$ for $D = \{0 \leq 7x + 5y \leq 1,\ 0 \leq -4x - 3y \leq 7\}$

1) 50.4

2) 28.

3) 81.2

4) 30.8

5) 33.6

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (4, 7)$.

1) $H(4, 7) = 5.15897$

2) $H(4, 7) = -4.25139$

3) $H(4, 7) = 0.5$

4) $H(4, 7) = -1.7297$

5) $H(4, 7) = -5.28069$

Exercise 4

Consider the vector field $F(x,y,z) = (-x, 0, -y)$ and the parameterized surface

$X: [0, 1] \times [-1, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2, 2uv, 2u^2 + 2v^2)$

Compute $\int_X F.$

1) 2.3 2) -0.1 3) 0. 4) -2.1

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 36

Exercise 1

Given the function

$f(x,y) = 3x^3 + 2y^3$ defined over the domain $D \equiv 9x^2 + 3y^2 \leq 39$, compute its absolute maxima and minima.

1) The value of the maximum is ****.4****

2) The value of the maximum is ****.0****

3) The value of the maximum is ****.7****

4) The value of the maximum is ****.5****

5) The value of the maximum is ****.1****

Exercise 2

Compute $\int_D (2x + y)\,dx\,dy$ for $D = \{0 \leq 3x - y \leq 2,\ 0 \leq -3x - 9y \leq 1\}$

1) 1.22778

2) 1.72778

3) 0.0277778

4) 0.327778

5) 0.427778

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (2, 5)$.

1) $H(2, 5) = -5.45221$

2) $H(2, 5) = -8.24197$

3) $H(2, 5) = 0.5$

4) $H(2, 5) = 8.66708$

5) $H(2, 5) = -3.41615$

Exercise 4

Consider the vector field $F(x,y,z) = (-y, -z, 0)$ and the parameterized surface

$X: [-1, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (uv + v^2, u, 1)$

Compute $\int_X F.$

1) 1.7 2) 1.8 3) 0. 4) -1.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 37

Exercise 1

Given the function

$f(x,y) = 3x^3 - y^3$ defined over the domain $D \equiv 27x^2 + 9y^2 \leq 1296$, compute its absolute maxima and minima.

1) The value of the maximum is ****.9****

2) The value of the maximum is ****.5****

3) The value of the maximum is ****.0****

4) The value of the maximum is ****.8****

5) The value of the maximum is ****.6****

Exercise 2

Compute $\int_D (2x + 2y)\,dx\,dy$ for $D = \{0 \leq -8x + 5y \leq 4,\ 0 \leq 6x - 4y \leq 2\}$

1) −131.1

2) −132.

3) −132.2

4) −130.2

5) −133.5

Exercise 3

Compute the mean curvature for $X(u,v) = \{2u, 1, v\}$ at the point $(u,v) = (2, 6)$.

1) $H(2, 6) = -5.21568$

2) $H(2, 6) = 8.1743$

3) $H(2, 6) = 0$

4) $H(2, 6) = -5.26533$

5) $H(2, 6) = 4.83323$

Exercise 4

Consider the vector field $F(x,y,z) = (0, 0, x)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (v, 4u^2, 4u)$

Compute $\int_X F.$

1) 0.8 2) −6.4 3) −1.6 4) −2.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 38

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{2x^2}{\sqrt{x^2+y^2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 32

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −47

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 1,\ z\ge 8\sqrt{x^2+y^2}\right\}$

1) 0.416173

2) 0.0161732

3) −1.88383

4) −0.483827

5) 0.916173

Exercise 3

Compute the center of curvature for $C(t)=\left\{2t^2,\ 3-t,\ -t^2\right\}$ at the point $t=-2$.

1) {39.9, −172.7, −137.7}

2) {−101.4, −47.1, −137.7}

3) {24.2, −157., −12.1}

4) {87., −235.5, −122.}

5) {55.6, −219.8, 129.2}

Exercise 4

Consider the vector field $F(x,y,z)=(-z,-y,0)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(1,2t^2,1+2t)$

Compute $\int_\sigma F$.

1) 2. 2) 8.8 3) −0.4 4) 6.2

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 39

Exercise 1

Given the function

$f(x,y) = -x^3 + 3y^3$ defined over the domain $D \equiv 6x^2 + 18y^2 \leq 384$, compute its absolute maxima and minima.

1) The value of the maximum is ****.3****

2) The value of the maximum is ****.1****

3) The value of the maximum is ****.8****

4) The value of the maximum is ****.6****

5) The value of the maximum is ****.0****

Exercise 2

Compute $\int_D (2x + 2y)\,dx\,dy$ for $D = \{0 \leq -5x - 9y \leq 3,\ 0 \leq 9x + y \leq 8\}$

1) 0.133241

2) 2.03324

3) 0.033241

4) 1.23324

5) -1.66676

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (0, 5)$.

1) $H(0, 5) = -1.32287$

2) $H(0, 5) = 8.28976$

3) $H(0, 5) = -7.56176$

4) $H(0, 5) = 8.06107$

5) $H(0, 5) = 0.5$

Exercise 4

Consider the vector field $F(x,y,z) = (-y, 0, 0)$ and the parameterized surface

$X: [-1, 1] \times [-1, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2uv, 2u^2, 2)$

Compute $\int_X F.$

1) 2.8 2) 2. 3) 0. 4) -0.3

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 40

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^4+y^4}{(x^2+y^2)^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -47

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -19

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 16,\ z\ge 15\sqrt{x^2+y^2}\right\}$

1) 0.29688

2) 1.19688

3) −1.30312

4) 2.29688

5) 0.0968803

Exercise 3

Compute the center of curvature for $C(t)=\left\{t^2, t, t^2\right\}$ at the point $t=-1$.

1) {0.05, 4., 8.85}

2) {2.45, 3.2, 5.65}

3) {0.05, 13.6, 0.85}

4) {3.25, 8., 3.25}

5) {10.45, 10.4, −1.55}

Exercise 4

Consider the vector field $F(x,y,z)=(z, x, 0)$ and the parameterized curve

$\sigma:[-1, 0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2t, 3, t+2t^2)$

Compute $\int_\sigma F$.

1) 0.333333 2) −2.86667 3) 0.233333 4) 2.23333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 41

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{2x^4 y}{(x^2+y^2)^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 25

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 49

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 169,\ z\ge 8\sqrt{x^2+y^2}\right\}$

1) 17.7662

2) 53.2987

3) 56.852

4) 0.

5) 35.5325

Exercise 3

Compute the center of curvature for $C(t)=\left\{2-t,\ 2t^2,\ -2t^2\right\}$ at the point $t=-1$.

1) {-30., 6.125, -6.125}

2) {-33., 15.125, 5.875}

3) {-15., 27.125, -9.125}

4) {-24., 30.125, 20.875}

5) {-45., -20.875, -12.125}

Exercise 4

Consider the vector field $F(x,y,z)=(0,0,-z)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2+2t^2,\ 2+2t^2,\ t+t^2)$

Compute $\int_\sigma F$.

1) -1.8 2) -3. 3) 0. 4) -1.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 42

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^3+3y^3}{-6x-3x^2+x^3+6x^4+3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 21

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 41

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\{x^2+y^2+z^2\le 144,\ z\ge x^2+y^2\}$

1) 217.028

2) 542.569

3) 651.083

4) -108.514

5) 21.7028

Exercise 3

Compute the center of curvature for $C(t)=\{-3t^2,\ -2t,\ -t^2\}$ at the point $t=-3$.

1) {188.4, -162., 188.8}

2) {-513.6, -972., 296.8}

3) {-243.6, -864., -405.2}

4) {-81.6, -540., -27.2}

5) {80.4, -216., -135.2}

Exercise 4

Consider the vector field $F(x,y,z)=(y,-x,0)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(t+t^2,2t^2,3t)$

Compute $\int_\sigma F$.

1) 6.46667 2) -1.33333 3) -7.93333 4) 0.266667

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 43

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^3+3y^3}{6x+9x^2+x^3-3x^4-3x^5-3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 21

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −18

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 36,\ z\ge 12\left(x^2+y^2\right)\right\}$

1) 0.

2) 3.27587

3) 13.1035

4) 4.67982

5) 0.467982

Exercise 3

Compute the center of curvature for $C(t)=\left\{-1+2t,\ 3t^2,\ -t^2\right\}$ at the point $t=1$.

1) {−6.3, 26.4, −5.3}

2) {−16.8, −7.2, 13.6}

3) {−21., 9.6, −3.2}

4) {−10.5, −3., 9.4}

5) {−35.7, 26.4, −1.1}

Exercise 4

Consider the vector field $F(x,y,z)=(0,z,-x)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2t^2,2t+t^2,2t)$

Compute $\int_\sigma F$.

1) −1.1 2) 4. 3) 0. 4) −1.5

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 44

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^3 - 2y^3}{2x + 3x^2 + x^3 + x^4 + x^5 - y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -17

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -37

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 100,\ z \ge 10\left(x^2 + y^2\right)\right\}$

1) 15.6297

2) 37.5112

3) -1.56297

4) -4.68891

5) 1.56297

Exercise 3

Compute the center of curvature for $C(t) = \left\{-3t^2, 2t, t^2\right\}$ at the point $t = -3$.

1) {26.4, 864., -404.8}

2) {-567.6, 972., -26.8}

3) {350.4, 810., 135.2}

4) {26.4, 270., 189.2}

5) {-81.6, 540., 27.2}

Exercise 4

Consider the vector field $F(x,y,z) = (y, x, 0)$ and the parameterized curve

$\sigma : [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (4, 2, t^2)$

Compute $\int_\sigma F$.

1) 1. 2) -1. 3) 0. 4) 2.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 45

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^2 y^2}{(x^2+y^2)^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 23

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 29

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 100,\ z\ge 3\sqrt{x^2+y^2}\right\}$

1) −96.7297

2) 107.477

3) −64.4865

4) 161.216

5) 118.225

Exercise 3

Compute the center of curvature for $C(t)=\left\{t^2,\ -2-3t,\ -t^2\right\}$ at the point $t=0$.

1) {2.25, −2., −2.25}

2) {0.45, −3., −1.65}

3) {0.85, −0.6, −2.65}

4) {0.85, −1., −0.45}

5) {0.85, −3.8, −1.85}

Exercise 4

Consider the vector field $F(x,y,z)=(0,x,0)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(1+2t^2,2t,t+t^2)$

Compute $\int_\sigma F$.

1) −1.76667 2) −0.866667 3) 3.33333 4) 6.63333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 46

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^2 + xy + 2y^2}{\sqrt{x^2+y^2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -48

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 43

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 144,\ z \ge 11\sqrt{x^2+y^2}\right\}$

1) 17.8355

2) 20.8081

3) 14.863

4) 29.7259

5) 29.7259

Exercise 3

Compute the center of curvature for $C(t) = \left\{1 + 3t,\ t^2,\ -2t^2\right\}$ at the point $t = 0$.

1) {0.2, 0.8, −2.}

2) {0.8, 1.2, −1.2}

3) {1., 0.9, −1.8}

4) {1.4, 0.3, −1.5}

5) {1.9, 0.3, −2.5}

Exercise 4

Consider the vector field $F(x,y,z) = (-z, y, 0)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2t, 2, 2)$

Compute $\int_\sigma F$.

1) −4. 2) 1.6 3) 9.6 4) −5.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 47

Exercise 1

Given the function

$f(x,y) = 5x^3 + 5y^3$ defined over the domain $D \equiv 15x^2 + 30y^2 \leq 540$, compute its absolute maxima and minima.

1) The value of the maximum is ****.3****

2) The value of the maximum is ****.5****

3) The value of the maximum is ****.6****

4) The value of the maximum is ****.1****

5) The value of the maximum is ****.0****

Exercise 2

Compute $\int_D (x + 2y)\,dx\,dy$ for $D = \{0 \leq 2x - 3y \leq 8,\ 0 \leq 9x - 5y \leq 4\}$

1) −6.63668

2) −8.63668

3) −8.13668

4) −10.1367

5) −8.33668

Exercise 3

Compute the mean curvature for $X(u,v) = \{v\,\mathrm{Cos}[u], v\,\mathrm{Sin}[u], v\}$ at the point $(u,v) = (2, 8)$.

1) $H(2, 8) = -7.94771$

2) $H(2, 8) = 2.38462$

3) $H(2, 8) = -8.65911$

4) $H(2, 8) = 5.72959$

5) $H(2, 8) = 0.0441942$

Exercise 4

Consider the vector field $F(x,y,z) = (0, y, 0)$ and the parameterized surface

$X: [0, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (4u, u^2, uv + v^2)$

Compute $\int_X F.$

1) −1.16667 2) 0.0333333 3) −0.466667 4) 0.333333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 48

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^4 - 2y^4}{6x + 9x^2 + x^4 - 3x^5 - 3x^6 - 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -55

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -33

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 225,\ z \geq 4\,(x^2 + y^2)\}$

1) 87.6251

2) 8.76251

3) 61.3375

4) −61.3375

5) −35.05

Exercise 3

Compute the center of curvature for $C(t) = \{2 - 2t,\ 2t^2,\ -t^2\}$ at the point $t = -3$.

1) {−509.2, 242.4, −268.6}

2) {−482.4, 188.8, −107.8}

3) {−268., 54.8, −27.4}

4) {−402., 162., −0.6}

5) {−26.8, 108.4, −107.8}

Exercise 4

Consider the vector field $F(x,y,z) = (0, y, 0)$ and the parameterized curve

$\sigma : [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (t + 2t^2, 2 + t, 1)$

Compute $\int_\sigma F$.

1) −6. 2) 10. 3) 4. 4) −2.4

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 49

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^3+3xy^2}{(x^2+y^2)^{3/2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 31

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 38

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 144,\ z\ge 2\sqrt{x^2+y^2}\right\}$

1) 191.04

2) −191.04

3) 1069.82

4) −191.04

5) 382.08

Exercise 3

Compute the center of curvature for $C(t)=\left\{3-t,\ 2t^2,\ -2t^2\right\}$ at the point $t=1$.

1) {35., 6.125, −6.125}

2) {66.5, 2.625, −2.625}

3) {10.5, −25.375, −13.125}

4) {49., 20.125, 14.875}

5) {59.5, 37.625, −9.625}

Exercise 4

Consider the vector field $F(x,y,z)=(0,y,-z)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(t+t^2,2+2t,2t^2)$

Compute $\int_\sigma F$.

1) 3.6 2) 17.6 3) −11.2 4) 4.

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Exam January-Call - hand for serial number: 50

Exercise 1

Given the function

$f(x,y) = -3x^3 + 3y^3$ defined over the domain $D \equiv 9x^2 + 9y^2 \leq 72$, compute its absolute maxima and minima.

1) The value of the minimum is ****.0****

2) The value of the minimum is ****.5****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.4****

5) The value of the minimum is ****.7****

Exercise 2

Compute $\int_D (3y)\,dx\,dy$ for $D = \{0 \leq -5x + 3y \leq 4,\ 0 \leq 2x - 8y \leq 6\}$

1) -2.08339

2) -1.38339

3) -2.18339

4) 0.116609

5) -1.18339

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (2, 5)$.

1) $H(2, 5) = -3.42978$

2) $H(2, 5) = 0.5$

3) $H(2, 5) = 8.50819$

4) $H(2, 5) = 7.61278$

5) $H(2, 5) = 7.84934$

Exercise 4

Consider the vector field $F(x,y,z) = (0, x, 0)$ and the parameterized surface

$X: [-1, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (uv, 2uv + 2v^2, 2)$

Compute $\int_X F.$

1) 2.9 2) -2.4 3) 0. 4) -0.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 51

Exercise 1

Given the function

$f(x,y) = 2x^3 + 3y^3$ defined over the domain $D \equiv 9x^2 + 18y^2 \leq 369$, compute its absolute maxima and minima.

1) The value of the minimum is ****.6****

2) The value of the minimum is ****.0****

3) The value of the minimum is ****.2****

4) The value of the minimum is ****.5****

5) The value of the minimum is ****.4****

Exercise 2

Compute $\int_D (4y)\,dx\,dy$ for $D = \{0 \leq -2x - 8y \leq 8,\ 0 \leq -2x - 3y \leq 4\}$

1) -5.12

2) -6.92

3) -3.12

4) -3.42

5) -4.12

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (1, 6)$.

1) $H(1, 6) = -4.10684$

2) $H(1, 6) = -3.38081$

3) $H(1, 6) = 6.15804$

4) $H(1, 6) = 0.5$

5) $H(1, 6) = 2.91099$

Exercise 4

Consider the vector field $F(x,y,z) = (y, 0, -x)$ and the parameterized surface

$X: [-1, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2u^2, u^2 + 2v^2, 2v + 2uv)$

Compute $\int_X F.$

1) -14.3 2) -3.3 3) 1.7 4) -4.8

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 52

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^4+2y^4}{2x+6x^2+x^4-2x^5-2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 19

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 5

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 25,\ z\ge 14\left(x^2+y^2\right)\right\}$

1) 4.73459

2) 3.89907

3) 1.39253

4) -0.557011

5) 2.78505

Exercise 3

Compute the center of curvature for $C(t)=\left\{3t^2,\ t^2,\ 1-2t\right\}$ at the point $t=2$.

1) {68.8, -116.6, 177.1}

2) {4.4, 124.9, 257.6}

3) {-92.2, -20., 241.5}

4) {36.6, 12.2, 161.}

5) {4.4, 44.4, 209.3}

Exercise 4

Consider the vector field $F(x,y,z)=(-z,0,x)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(t^2,t+2t^2,2)$

Compute $\int_\sigma F$.

1) -2.5 2) 0. 3) 0.6 4) -1.2

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 53

Exercise 1

Given the function

$f(x,y) = 3x^3 + 3y^3$ defined over the domain $D \equiv 18x^2 + 27y^2 \leq 1260$, compute its absolute maxima and minima.

1) The value of the maximum is ****.9****

2) The value of the maximum is ****.8****

3) The value of the maximum is ****.0****

4) The value of the maximum is ****.5****

5) The value of the maximum is ****.3****

Exercise 2

Compute $\int_D (3x + 2y)\,dx\,dy$ for $D = \{0 \leq 2x + 3y \leq 3,\ 0 \leq -3x - y \leq 4\}$

1) -0.646939

2) -0.146939

3) -0.646939

4) 0.653061

5) -1.34694

Exercise 3

Compute the mean curvature for $X(u,v) = \{e^v \cos[u], e^v \sin[u], v\}$ at the point $(u,v) = (5, 10)$.

1) $H(5, 10) = -3.60744$

2) $H(5, 10) = -7.46227$

3) $H(5, 10) = -3.46286$

4) $H(5, 10) = -7.44209$

5) $H(5, 10) = 0$

Exercise 4

Consider the vector field $F(x,y,z) = (-z, 0, x)$ and the parameterized surface

$X: [0, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v, v^2, u^2)$

Compute $\int_X F.$

1) 0.2 2) -1.1 3) -0.5 4) -1.5

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 54

Exercise 1

Given the function

$f(x,y) = 4x^3 + 5y^3$ defined over the domain $D \equiv 12x^2 + 45y^2 \leq 1668$, compute its absolute maxima and minima.

1) The value of the maximum is ****.9****

2) The value of the maximum is ****.8****

3) The value of the maximum is ****.1****

4) The value of the maximum is ****.2****

5) The value of the maximum is ****.7****

Exercise 2

Compute $\int_D (2x + y)\,dx\,dy$ for $D = \{0 \leq -2x - 8y \leq 5,\ 0 \leq 6x + 8y \leq 6\}$

1) 5.49609

2) 1.96289

3) 4.90723

4) 5.49609

5) 4.12207

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (6, 8)$.

1) $H(6, 8) = 8.51977$

2) $H(6, 8) = 4.591$

3) $H(6, 8) = 0.5$

4) $H(6, 8) = 8.2838$

5) $H(6, 8) = 3.69649$

Exercise 4

Consider the vector field $F(x,y,z) = (-y, z, 0)$ and the parameterized surface

$X: [-1, 0] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1, v + 2uv, u)$

Compute $\int_X F.$

1) 0.166667 2) −2.63333 3) 3.56667 4) −2.53333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 55

Exercise 1

Given the function

$f(x,y) = -4x^3 + 5y^3$ defined over the domain $D \equiv 18x^2 + 45y^2 \leq 1782$, compute its absolute maxima and minima.

1) The value of the maximum is ****.6****

2) The value of the maximum is ****.5****

3) The value of the maximum is ****.7****

4) The value of the maximum is ****.1****

5) The value of the maximum is ****.0****

Exercise 2

Compute $\int_D (4y)\,dx\,dy$ for $D = \{0 \leq -7x + 2y \leq 7,\ 0 \leq -7x - 3y \leq 1\}$

1) 1.68

2) -0.22

3) 1.98

4) 1.98

5) 0.48

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (3, 9)$.

1) $H(3, 9) = -8.44226$

2) $H(3, 9) = 7.47597$

3) $H(3, 9) = 1.55358$

4) $H(3, 9) = -8.69018$

5) $H(3, 9) = 0.5$

Exercise 4

Consider the vector field $F(x,y,z) = (0, -x, -z)$ and the parameterized surface

$X: [-1, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v^2, u, 1 + u)$

Compute $\int_X F.$

1) 1.4 2) 0. 3) -1. 4) -1.3

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 56

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^3 - 2y^3}{3x + 9x^2 + x^3 - 6x^4 - 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 7

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 1

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 196,\ z \geq 8\ (x^2 + y^2)\}$

1) 22.9879

2) 80.4578

3) 11.494

4) 38.3132

5) -38.3132

Exercise 3

Compute the center of curvature for $C(t) = \{-2t^2,\ 1 - 3t,\ 3t^2\}$ at the point $t = -2$.

1) {30.1077, -28.0667, -17.7615}

2) {-79.4923, -206.167, -31.4615}

3) {-65.7923, -165.067, -45.1615}

4) {2.70769, -247.267, -17.7615}

5) {-24.6923, -137.667, 37.0385}

Exercise 4

Consider the vector field $F(x,y,z) = (-y, -z, 0)$ and the parameterized curve

$\sigma: [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (4t^2, 3t, t^2)$

Compute $\int_\sigma F$.

1) -7.2 2) -32.4 3) -18. 4) -88.2

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 57

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3y - 3x^2y^2}{(x^2+y^2)^{3/2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 37

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −42

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 4,\ z\ge 2\sqrt{x^2+y^2}\right\}$

1) 2.83022

2) 2.47645

3) 3.89156

4) 1.76889

5) −0.707556

Exercise 3

Compute the center of curvature for $C(t)=\left\{-2-2t,\ 2t^2,\ -2t^2\right\}$ at the point $t=-3$.

1) {−781.2, 228.1, 119.1}

2) {−390.6, −292.7, −314.9}

3) {−173.6, 97.9, −271.5}

4) {−434., 54.5, −54.5}

5) {−607.6, 228.1, 249.3}

Exercise 4

Consider the vector field $F(x,y,z)=(z,y,0)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2,2,2t^2)$

Compute $\int_\sigma F$.

1) 3.7 2) 3.3 3) 0. 4) −1.5

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 58

Exercise 1

Given the function

$f(x,y) = 4x^3 - 5y^3$ defined over the domain $D \equiv 30x^2 + 45y^2 \leq 2370$, compute its absolute maxima and minima.

1) The value of the maximum is ****.7****

2) The value of the maximum is ****.3****

3) The value of the maximum is ****.2****

4) The value of the maximum is ****.6****

5) The value of the maximum is ****.0****

Exercise 2

Compute $\int_D (2x + y)\,dx\,dy$ for $D = \{0 \leq -2x - 3y \leq 3,\ 0 \leq 9x - 5y \leq 2\}$

1) 1.79262

2) -1.10738

3) -0.107378

4) 1.79262

5) -0.707378

Exercise 3

Compute the mean curvature for $X(u,v) = \{e^v \cos[u], e^v \sin[u], v\}$ at the point $(u,v) = (5, 7)$.

1) $H(5, 7) = -0.773$

2) $H(5, 7) = 3.31416$

3) $H(5, 7) = -6.18431$

4) $H(5, 7) = 0.631865$

5) $H(5, 7) = 0$

Exercise 4

Consider the vector field $F(x,y,z) = (0, -y, -z)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v, 2uv, v)$

Compute $\int_X F.$

1) -1.36667 2) 0.433333 3) 1.33333 4) 3.83333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 59

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{-6x-6x^2+x^3+6x^4-6x^5+3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 13

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 9

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 225,\ z\ge 13\left(x^2+y^2\right)\right\}$

1) 29.829

2) 27.1173

3) 48.8111

4) 13.5586

5) 37.9642

Exercise 3

Compute the center of curvature for $C(t)=\left\{t^2,\ t^2,\ -3+3t\right\}$ at the point $t=3$.

1) {81.75, 96.75, −37.5}

2) {21.75, −0.75, −120.}

3) {36.75, 44.25, −60.}

4) {29.25, 29.25, −75.}

5) {6.75, 81.75, −97.5}

Exercise 4

Consider the vector field $F(x,y,z)=(-x,0,-z)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(t+2t^2,\ t+t^2,\ 1)$

Compute $\int_\sigma F$.

1) −2.4 2) −11.2 3) −11.6 4) −4.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 60

Exercise 1

Given the function

$f(x,y) = 2x^3 + 5y^3$ defined over the domain $D \equiv 9x^2 + 45y^2 \leq 1701$, compute its absolute maxima and minima.

1) The value of the maximum is ****.3****

2) The value of the maximum is ****.4****

3) The value of the maximum is ****.9****

4) The value of the maximum is ****.7****

5) The value of the maximum is ****.6****

Exercise 2

Compute $\int_D (3y)\,dx\,dy$ for $D = \{0 \leq 4x - 3y \leq 7,\ 0 \leq -2x - 9y \leq 6\}$

1) −1.35714

2) 0.642857

3) −0.0571429

4) 0.0428571

5) −0.157143

Exercise 3

Compute the mean curvature for $X(u,v) = \{u, u, v\}$ at the point $(u,v) = (5, 7)$.

1) $H(5, 7) = -5.68088$

2) $H(5, 7) = -1.52123$

3) $H(5, 7) = -1.8185$

4) $H(5, 7) = 6.64916$

5) $H(5, 7) = 0$

Exercise 4

Consider the vector field $F(x,y,z) = (x, -z, -y)$ and the parameterized surface

$X: [0, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2u^2, 2uv, 2u^2 + 2uv)$

Compute $\int_X F.$

1) 0.4 2) −0.5 3) −2.2 4) 0.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 61

Exercise 1

Given the function

$f(x,y) = 5x^3 - y^3$ defined over the domain $D \equiv 30x^2 + 3y^2 \leq 492$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.2****

3) The value of the minimum is ****.3****

4) The value of the minimum is ****.4****

5) The value of the minimum is ****.6****

Exercise 2

Compute $\int_D (6x)\,dx\,dy$ for $D = \{0 \leq -8x - 3y \leq 2,\ 0 \leq 2x - 7y \leq 8\}$

1) -0.97513

2) -1.27513

3) 1.42487

4) 0.12487

5) 0.82487

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (5, 1)$.

1) $H(5, 1) = 5.43922$

2) $H(5, 1) = 1.79078$

3) $H(5, 1) = 0.5$

4) $H(5, 1) = 5.76316$

5) $H(5, 1) = -7.93199$

Exercise 4

Consider the vector field $F(x,y,z) = (z, 0, 0)$ and the parameterized surface

$X: [-1, 0] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1 + u^2, 2uv, 2u + 2uv)$

Compute $\int_X F.$

1) -1.33333 2) 1.86667 3) -3.13333 4) -0.533333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 62

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^4 - 2y^4}{-3x - 3x^2 + x^4 + x^5 + x^6 + y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -255

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -160

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 25,\ z \ge 4\left(x^2 + y^2\right)\right\}$

1) -4.78805

2) 22.9826

3) 22.9826

4) -6.70326

5) 9.57609

Exercise 3

Compute the center of curvature for $C(t) = \left\{-t^2, -2 - 2t, -2t^2\right\}$ at the point $t = -3$.

1) $\{-136.2, -190.4, 108.4\}$

2) $\{-0.2, -408., -0.4\}$

3) $\{-190.6, -163.2, -136.4\}$

4) $\{190.2, -408., -272.4\}$

5) $\{-27.4, -272., -54.8\}$

Exercise 4

Consider the vector field $F(x,y,z) = (0, y, 0)$ and the parameterized curve

$\sigma : [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (t^2, 2t, 2t)$

Compute $\int_\sigma F$.

1) $0.$ 2) -3.8 3) -1.7 4) $3.$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 63

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2x^3 - 2x^2 y}{(x^2+y^2)^{3/2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -48

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -28

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 9,\ z \ge 13\sqrt{x^2+y^2}\right\}$

1) 1.96656

2) 0.166565

3) −0.333435

4) −0.0334351

5) −0.933435

Exercise 3

Compute the center of curvature for $C(t) = \{3 - 3t,\ 3t^2,\ -3t^2\}$ at the point $t = 1$.

1) {8.1, −3.75, 14.55}

2) {40.5, 17.85, −7.05}

3) {27., 9.75, −9.75}

4) {10.8, 25.95, 14.55}

5) {18.9, 15.15, −12.45}

Exercise 4

Consider the vector field $F(x,y,z) = (0, y, 0)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (t, 1 + t^2, 2t^2)$

Compute $\int_\sigma F$.

1) −1.5 2) 0.3 3) 4.5 4) 6.3

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 64

Exercise 1

Given the function

$f(x,y) = 5x^3 + 5y^3$ defined over the domain $D \equiv 45x^2 + 15y^2 \leq 1680$, compute its absolute maxima and minima.

1) The value of the minimum is ****.4****

2) The value of the minimum is ****.2****

3) The value of the minimum is ****.9****

4) The value of the minimum is ****.7****

5) The value of the minimum is ****.3****

Exercise 2

Compute $\int_D (3x + 2y)\,dx\,dy$ for $D = \{0 \leq 9x - y \leq 1,\ 0 \leq -6x - 3y \leq 7\}$

1) -2.48209

2) -0.482094

3) -0.0820937

4) -1.68209

5) 0.217906

Exercise 3

Compute the mean curvature for $X(u,v) = \{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v) = (2, 2)$.

1) $H(2, 2) = 5.7405$

2) $H(2, 2) = -5.21778$

3) $H(2, 2) = 3.079$

4) $H(2, 2) = 0.176777$

5) $H(2, 2) = -0.911611$

Exercise 4

Consider the vector field $F(x,y,z) = (z, x, 0)$ and the parameterized surface

$X: [0, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (v^2, v^2, u)$

Compute $\int_X F.$

1) 3.9 2) -3.7 3) 0. 4) -0.5

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 65

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^3+3xy^2}{x^2+y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -37

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -7

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 36,\ z\ge 7\sqrt{x^2+y^2}\right\}$

1) 4.54674

2) 7.72946

3) 13.1856

4) 3.18272

5) 7.72946

Exercise 3

Compute the center of curvature for $C(t)=\left\{-t^2,\ 2+t,\ t^2\right\}$ at the point $t=2$.

1) {-12.25, -62., 12.25}

2) {37.35, -31., -0.15}

3) {31.15, -31., -6.35}

4) {24.95, -68.2, 61.85}

5) {0.15, -74.4, 55.65}

Exercise 4

Consider the vector field $F(x,y,z)=(0,y,z)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2t^2,1,4t^2)$

Compute $\int_\sigma F$.

1) -32.8 2) -27.2 3) -8. 4) -19.2

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 66

Exercise 1

Given the function

$f(x,y) = -2x^3 + 5y^3$ defined over the domain $D \equiv 12x^2 + 15y^2 \leq 252$, compute its absolute maxima and minima.

1) The value of the maximum is ****.4****

2) The value of the maximum is ****.2****

3) The value of the maximum is ****.6****

4) The value of the maximum is ****.1****

5) The value of the maximum is ****.5****

Exercise 2

Compute $\int_D (3y)\,dx\,dy$ for $D = \{0 \leq -3x + 4y \leq 5,\ 0 \leq -9x - 3y \leq 9\}$

1) 1.3

2) -1.3

3) -0.4

4) 2.4

5) 0.6

Exercise 3

Compute the mean curvature for $X(u,v) = \{3u, 2u, v\}$ at the point $(u,v) = (4,4)$.

1) $H(4,4) = 0$

2) $H(4,4) = 2.50278$

3) $H(4,4) = 4.11744$

4) $H(4,4) = -5.35821$

5) $H(4,4) = -4.43052$

Exercise 4

Consider the vector field $F(x,y,z) = (y, z, 0)$ and the parameterized surface

$X: [-1, 0] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (u + v^2, 2 + u^2, 1 + 2u)$

Compute $\int_X F$.

1) -3.7 2) -3.1 3) 0. 4) -3.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 67

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^2+3y^2}{x^2+y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -3

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 23

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 100,\ z\geq 3\sqrt{x^2+y^2}\right\}$

1) -32.2432

2) 107.477

3) -107.477

4) 236.45

5) -10.7477

Exercise 3

Compute the center of curvature for $C(t)=\left\{2+3t,\ 3t^2,\ 3t^2\right\}$ at the point $t=-1$.

1) {13., 12.35, 22.75}

2) {31.2, 14.95, 20.15}

3) {31.2, 20.15, 20.15}

4) {31.2, 17.55, −8.45}

5) {26., 9.75, 9.75}

Exercise 4

Consider the vector field $F(x,y,z)=(0,0,z)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(1+2t^2,2t^2,2t+2t^2)$

Compute $\int_\sigma F$.

1) $-4.$ 2) 3.5 3) $-2.$ 4) $0.$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 68

Exercise 1

Given the function

$f(x,y) = 3x^3 + y^3$ defined over the domain $D \equiv 18x^2 + 6y^2 \leq 384$, compute its absolute maxima and minima.

1) The value of the maximum is ****.2****

2) The value of the maximum is ****.0****

3) The value of the maximum is ****.5****

4) The value of the maximum is ****.8****

5) The value of the maximum is ****.6****

Exercise 2

Compute $\int_D (3x + 2y)\,dx\,dy$ for $D = \{0 \leq 3x - 6y \leq 6,\ 0 \leq 2x + 7y \leq 4\}$

1) 2.18182

2) 5.01818

3) 5.89091

4) 3.27273

5) 3.92727

Exercise 3

Compute the mean curvature for $X(u,v) = \{2u, 2u^2, v\}$ at the point $(u,v) = (1, 6)$.

1) $H(1, 6) = 3.70279$

2) $H(1, 6) = -5.08921$

3) $H(1, 6) = 0.646913$

4) $H(1, 6) = -3.82714$

5) $H(1, 6) = 0.0447214$

Exercise 4

Consider the vector field $F(x,y,z) = (z, 0, -y)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2 + 2uv, 3u, 1 + v^2)$

Compute $\int_X F.$

1) 10.5 2) 22.5 3) 46.5 4) -23.5

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 69

Exercise 1

Given the function

$f(x,y) = -4x^3 - 5y^3$ defined over the domain $D \equiv 30x^2 + 15y^2 \leq 810$, compute its absolute maxima and minima.

1) The value of the maximum is ****.1****

2) The value of the maximum is ****.0****

3) The value of the maximum is ****.7****

4) The value of the maximum is ****.9****

5) The value of the maximum is ****.8****

Exercise 2

Compute $\int_D (2x + 3y)\,dx\,dy$ for $D = \{0 \leq -8x + 3y \leq 1,\ 0 \leq -8x - 5y \leq 6\}$

1) -0.921582

2) -0.521582

3) -0.121582

4) 0.978418

5) 0.378418

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (3, 10)$.

1) $H(3, 10) = 0.5$

2) $H(3, 10) = 3.83601$

3) $H(3, 10) = -4.79905$

4) $H(3, 10) = -8.36429$

5) $H(3, 10) = -5.28166$

Exercise 4

Consider the vector field $F(x,y,z) = (0, -x, -y)$ and the parameterized surface

$X: [-1, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2u^2, 1, u^2 + uv)$

Compute $\int_X F.$

1) 12.2 2) 3.2 3) 15.2 4) 10.7

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 70

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{x\,y}{\sqrt{x^2+y^2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -26

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 19

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 225,\ z\ge 8\sqrt{x^2+y^2}\right\}$

1) -54.5845

2) 54.5845

3) 38.2091

4) 70.9598

5) 49.126

Exercise 3

Compute the center of curvature for $C(t)=\left\{-1-t,\ 3\,t^2,\ -t^2\right\}$ at the point $t=-3$.

1) $\{-1513.4,\ -567.45,\ -567.55\}$

2) $\{-540.5,\ 189.25,\ 405.35\}$

3) $\{-972.9,\ -567.45,\ 837.75\}$

4) $\{-1081.,\ 81.15,\ -27.05\}$

5) $\{-1297.2,\ -567.45,\ 621.55\}$

Exercise 4

Consider the vector field $F(x,y,z)=(-z,\,-y,\,-x)$ and the parameterized curve

$\sigma:[-1,\,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(1+t^2,\,2+2\,t,\,1)$

Compute $\int_\sigma F$.

1) -2.4 2) -2.9 3) $-1.$ 4) 2.9

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 71

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^4 - y^4}{6x + 3x^2 + x^4 - 3x^5 - 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -20

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -17

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 9,\ z \ge 15\left(x^2 + y^2\right)\right\}$

1) 0.832083

2) 0.532083

3) 0.232083

4) -0.0679169

5) 0.932083

Exercise 3

Compute the center of curvature for $C(t) = \left\{3t^2, 3t^2, 1 + t\right\}$ at the point $t = -1$.

1) {-34.7167, -49.3167, 138.7}

2) {9.08333, 9.08333, 73.}

3) {-12.8167, 45.5833, 65.7}

4) {1.78333, 16.3833, 65.7}

5) {60.1833, 38.2833, 102.2}

Exercise 4

Consider the vector field $F(x,y,z) = (0, -x, y)$ and the parameterized curve

$\sigma : [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (t, t + t^2, 1)$

Compute $\int_\sigma F$.

1) -1.33333 2) 0.266667 3) 6.06667 4) 5.86667

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 72

Exercise 1

Given the function

$f(x,y) = -4x^3 + 2y^3$ defined over the domain $D \equiv 24x^2 + 9y^2 \leq 465$, compute its absolute maxima and minima.

1) The value of the minimum is ****.6****

2) The value of the minimum is ****.5****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.7****

Exercise 2

Compute $\int_D (5x)\,dx\,dy$ for $D = \{0 \leq 6x + 4y \leq 2,\ 0 \leq -3x + 9y \leq 7\}$

1) −2.08035

2) 0.819651

3) −1.08035

4) 0.219651

5) −0.0803489

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \mathrm{Cos}[u],\ v^2 \mathrm{Sin}[u],\ v\}$ at the point $(u,v) = (4, 7)$.

1) $H(4, 7) = 3.38306$

2) $H(4, 7) = 0.777315$

3) $H(4, 7) = -1.82428$

4) $H(4, 7) = 0.000365351$

5) $H(4, 7) = 8.65351$

Exercise 4

Consider the vector field $F(x,y,z) = (0, z, x)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (u^2 + 2uv, 2u^2 + v, 3v^2)$

Compute $\int_X F.$

1) 6.46667 2) −10.3333 3) 17.6667 4) −13.1333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 73

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^4+3y^4}{3x+2x^2+x^4+2x^5-y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 221

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 246

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 4,\ z\geq 10\left(x^2+y^2\right)\right\}$

1) 2.11287

2) -1.18713

3) 1.91287

4) 2.21287

5) 0.61287

Exercise 3

Compute the center of curvature for $C(t)=\left\{-3t^2,\ 2t^2,\ 2-3t\right\}$ at the point $t=0$.

1) {0.761538, 1.49231, 1.2}

2) {0.561538, 0.492308, 3.2}

3) {-1.03846, 0.692308, 2.}

4) {-0.638462, 2.09231, 1.4}

5) {-2.63846, 2.49231, 1.8}

Exercise 4

Consider the vector field $F(x,y,z)=(0,y,-z)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2,2+2t^2,2)$

Compute $\int_\sigma F$.

1) 2.9 2) 3.8 3) 0. 4) -0.5

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 74

Exercise 1

Given the function

$f(x,y) = -4x^3 - y^3$ defined over the domain $D \equiv 12x^2 + 6y^2 \leq 144$, compute its absolute maxima and minima.

1) The value of the maximum is ****.0****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.9****

4) The value of the maximum is ****.2****

5) The value of the maximum is ****.8****

Exercise 2

Compute $\int_D (2x + y)\,dx\,dy$ for $D = \{0 \leq 7x + y \leq 6,\ 0 \leq 4x + 2y \leq 3\}$

1) -0.135

2) 1.35

3) 3.24

4) -0.27

5) 1.08

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \cos[u], v^2 \sin[u], v\}$ at the point $(u,v) = (1, 6)$.

1) $H(1, 6) = 6.77161$

2) $H(1, 6) = 6.40527$

3) $H(1, 6) = 0.000580682$

4) $H(1, 6) = -3.16915$

5) $H(1, 6) = -8.81513$

Exercise 4

Consider the vector field $F(x,y,z) = (0, -y, z)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1 + 2uv, u + 2uv, u + uv)$

Compute $\int_X F.$

1) -1.13333 2) -9.83333 3) 1.26667 4) -2.33333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 75

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^4+3y^4}{-9x-3x^2+x^4+3x^5+3x^6+3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -342

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 246

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 36,\ z\geq 3\left(x^2+y^2\right)\right\}$

1) 16.502

2) 18.3356

3) 29.3369

4) 40.3382

5) 27.5033

Exercise 3

Compute the center of curvature for $C(t)=\left\{-2t^2,\ -2-2t,\ -t^2\right\}$ at the point $t=-2$.

1) {8., −49.2, 20.4}

2) {8., −65.6, 53.2}

3) {−8.4, −73.8, −53.4}

4) {−24.8, −82., −12.4}

5) {−98.6, −49.2, 12.2}

Exercise 4

Consider the vector field $F(x,y,z)=(z,0,x)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2t^2,t,2t^2)$

Compute $\int_\sigma F$.

1) 0.3 2) 0. 3) −4. 4) −3.5

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 76

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^4+y^4}{-4x-2x^2+x^4+2x^5+2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 15

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 26

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 100,\ z\ge 3\left(x^2+y^2\right)\right\}$

1) 46.3472

2) 30.8981

3) 87.5446

4) 51.4968

5) -10.2994

Exercise 3

Compute the center of curvature for $C(t)=\left\{t^2,\ -t^2,\ -3-t\right\}$ at the point $t=2$.

1) {18.35, -36.65, 79.3}

2) {12.25, -12.25, 61.}

3) {18.35, -67.15, 115.9}

4) {18.35, -36.65, 18.3}

5) {6.15, -18.35, 30.5}

Exercise 4

Consider the vector field $F(x,y,z)=(x,0,0)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(1+2t^2,t,3t)$

Compute $\int_\sigma F$.

1) -1.2 2) -2.7 3) 2.2 4) 0.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 77

Exercise 1

Given the function

$f(x,y) = 2x^3 + 4y^3$ defined over the domain $D \equiv 3x^2 + 12y^2 \leq 51$, compute its absolute maxima and minima.

1) The value of the maximum is ****.4****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.2****

4) The value of the maximum is ****.9****

5) The value of the maximum is ****.1****

Exercise 2

Compute $\int_D (4y)\,dx\,dy$ for $D = \{0 \leq -x - 5y \leq 7,\ 0 \leq -5x + 9y \leq 8\}$

1) -2.61592

2) -1.31592

3) -0.815917

4) -1.41592

5) -1.51592

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (1, 8)$.

1) $H(1, 8) = 5.67307$

2) $H(1, 8) = -6.14992$

3) $H(1, 8) = -6.46292$

4) $H(1, 8) = -8.19927$

5) $H(1, 8) = 0.5$

Exercise 4

Consider the vector field $F(x,y,z) = (y, 0, 0)$ and the parameterized surface

$X: [-1, 1] \times [-1, 0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v + 2v^2, u + u^2, 2u^2 + v)$

Compute $\int_X F.$

1) -6. 2) 2. 3) 5. 4) 9.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 78

Exercise 1

Given the function

$f(x,y) = -5\,x^3 - 5\,y^3$ defined over the domain $D \equiv 45\,x^2 + 30\,y^2 \leq 2100$, compute its absolute maxima and minima.

1) The value of the maximum is ****.3****

2) The value of the maximum is ****.1****

3) The value of the maximum is ****.6****

4) The value of the maximum is ****.5****

5) The value of the maximum is ****.2****

Exercise 2

Compute $\int_D (4\,y)\,dx\,dy$ for $D = \{0 \leq 9\,x + 5\,y \leq 1,\ 0 \leq 3\,x - y \leq 7\}$

1) -2.15833

2) -0.958333

3) -2.05833

4) -1.45833

5) -2.05833

Exercise 3

Compute the mean curvature for $X(u,v) = \{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v) = (2,2)$.

1) $H(2,2) = -0.35128$

2) $H(2,2) = -0.886953$

3) $H(2,2) = 0.176777$

4) $H(2,2) = 2.15965$

5) $H(2,2) = -6.98596$

Exercise 4

Consider the vector field $F(x,y,z) = (-x, 0, -y)$ and the parameterized surface

$X: [0,1] \times [-1,0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2\,u, 2\,u\,v, v)$

Compute $\int_X F.$

1) -2.06667 2) 3.13333 3) -1.46667 4) 2.33333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 79

Exercise 1

Given the function

$f(x,y) = -4x^3 - 4y^3$ defined over the domain $D \equiv 24x^2 + 18y^2 \leq 546$, compute its absolute maxima and minima.

1) The value of the minimum is ****.1****

2) The value of the minimum is ****.6****

3) The value of the minimum is ****.0****

4) The value of the minimum is ****.2****

5) The value of the minimum is ****.3****

Exercise 2

Compute $\int_D (5x)\,dx\,dy$ for $D = \{0 \leq -9x + 6y \leq 6,\ 0 \leq 8x - y \leq 1\}$

1) -0.381657

2) -1.88166

3) 0.118343

4) 0.0183432

5) -0.681657

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \mathrm{Cos}[u],\ v^2 \mathrm{Sin}[u],\ v\}$ at the point $(u,v) = (6,5)$.

1) $H(6,5) = 0.00100489$

2) $H(6,5) = -4.96021$

3) $H(6,5) = -8.40302$

4) $H(6,5) = 2.62175$

5) $H(6,5) = 2.41086$

Exercise 4

Consider the vector field $F(x,y,z) = (y, 0, x)$ and the parameterized surface

$X: [0,1] \times [0,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (3u^2, uv + 2v^2, 1 + v^2)$

Compute $\int_X F.$

1) 13.65 2) 4.55 3) -5.85 4) -38.35

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 80

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^2y^2 - 2xy^3 - y^4}{(x^2+y^2)^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -18

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -31

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2+y^2+z^2 \le 9,\ z \ge 8\sqrt{x^2+y^2}\right\}$

1) 0.536676

2) −0.0633242

3) 0.0366758

4) 0.436676

5) −1.46332

Exercise 3

Compute the center of curvature for $C(t) = \{-2t^2, 3t^2, -3+t\}$ at the point $t = -2$.

1) {−271.877, −5.18462, 165.2}

2) {141.123, 77.4154, 82.6}

3) {−189.277, 325.215, 247.8}

4) {−147.977, −5.18462, 660.8}

5) {−24.0769, 36.1154, 413.}

Exercise 4

Consider the vector field $F(x,y,z) = (z, -x, 0)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (t, 2t^2, 1+t^2)$

Compute $\int_\sigma F$.

1) −3.4 2) 0. 3) 0.7 4) 1.3

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 81

Exercise 1

Given the function

$f(x,y) = 5x^3 - 4y^3$ defined over the domain $D \equiv 30x^2 + 24y^2 \leq 864$, compute its absolute maxima and minima.

1) The value of the maximum is ****.4****

2) The value of the maximum is ****.0****

3) The value of the maximum is ****.5****

4) The value of the maximum is ****.8****

5) The value of the maximum is ****.7****

Exercise 2

Compute $\int_D (3x)\,dx\,dy$ for $D = \{0 \leq 5x \leq 5,\ 0 \leq 6x - 5y \leq 4\}$

1) 1.2

2) 1.92

3) −0.72

4) −1.2

5) 3.24

Exercise 3

Compute the mean curvature for $X(u,v) = \{e^v \text{Cos}[u],\ e^v \text{Sin}[u],\ v\}$ at the point $(u,v) = (2, 10)$.

1) $H(2, 10) = 8.52533$

2) $H(2, 10) = -5.74107$

3) $H(2, 10) = 5.0411$

4) $H(2, 10) = 0$

5) $H(2, 10) = -1.11327$

Exercise 4

Consider the vector field $F(x,y,z) = (-z, 0, y)$ and the parameterized surface

$X: [0, 1] \times [-1, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2u + 2u^2,\ 2u^2 + 2uv,\ 2 + 2v)$

Compute $\int_X F.$

1) −10.9333 2) 28.6667 3) −9.83333 4) −39.5333

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Exercise 1

Given the function

$f(x,y) = 4x^3 - 3y^3$ defined over the domain $D \equiv 6x^2 + 18y^2 \leq 294$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.2****

3) The value of the minimum is ****.1****

4) The value of the minimum is ****.0****

5) The value of the minimum is ****.7****

Exercise 2

Compute $\int_D (4y)\,dx\,dy$ for $D = \{0 \leq -2x + 9y \leq 8,\ 0 \leq 4x - 9y \leq 5\}$

1) −7.25926

2) 18.6667

3) 28.

4) 10.3704

5) 3.11111

Exercise 3

Compute the mean curvature for $X(u,v) = \{e^v \text{Cos}[u], e^v \text{Sin}[u], v\}$ at the point $(u,v) = (5, 3)$.

1) $H(5, 3) = -7.8239$

2) $H(5, 3) = 5.86422$

3) $H(5, 3) = -7.19018$

4) $H(5, 3) = 3.06072 \times 10^{-6}$

5) $H(5, 3) = -1.15815$

Exercise 4

Consider the vector field $F(x,y,z) = (y, x, -z)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2uv, 2, 1 + u)$

Compute $\int_X F.$

1) −2.53333 2) 0.666667 3) 3.46667 4) −0.533333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 83

Exercise 1

Given the function

$f(x,y) = 3x^3 + 4y^3$ defined over the domain $D \equiv 18x^2 + 12y^2 \leq 336$, compute its absolute maxima and minima.

1) The value of the maximum is ****. 2 ****

2) The value of the maximum is ****. 5 ****

3) The value of the maximum is ****. 7 ****

4) The value of the maximum is ****. 8 ****

5) The value of the maximum is ****. 6 ****

Exercise 2

Compute $\int_D (2x + y)\,dx\,dy$ for $D = \{0 \leq -3x + 6y \leq 3,\ 0 \leq 4x + 4y \leq 2\}$

1) 0.0416667

2) 0.941667

3) 0.441667

4) 1.24167

5) 1.44167

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (2, 7)$.

1) $H(2, 7) = -5.24362$

2) $H(2, 7) = -4.24766$

3) $H(2, 7) = -2.08441$

4) $H(2, 7) = -2.67039$

5) $H(2, 7) = 0.5$

Exercise 4

Consider the vector field $F(x,y,z) = (0, 0, y)$ and the parameterized surface

$X: [-1, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2uv, 2u + v, 4u^2)$

Compute $\int_X F$.

1) −11.2 2) −4. 3) −10.8 4) −11.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 84

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^4 - y^4}{-9x - 9x^2 + x^4 + 3x^5 + 3x^6 + 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -82

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -160

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 64,\ z \ge 13\left(x^2 + y^2\right)\right\}$

1) 7.69609

2) −7.69609

3) 2.30883

4) −5.38726

5) 8.4657

Exercise 3

Compute the center of curvature for $C(t) = \left\{-2 - t,\ -t^2,\ t^2\right\}$ at the point $t = 2$.

1) {24.8, 12.55, −0.15}

2) {74.4, 6.35, 18.45}

3) {6.2, −68.05, 55.65}

4) {49.6, −18.45, −12.55}

5) {62., −12.25, 12.25}

Exercise 4

Consider the vector field $F(x,y,z) = (z, x, 0)$ and the parameterized curve

$\sigma : [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (t^2, 1 + t^2, 2t^2)$

Compute $\int_\sigma F$.

1) 0. 2) 1.7 3) −4. 4) 1.1

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 85

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^4 - 3y^4}{-4x - 4x^2 + x^4 - 2x^5 + 4x^6 + 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -49

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -45

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 9,\ z \ge 4\left(x^2 + y^2\right)\right\}$

1) 1.69553

2) 8.81675

3) 2.03463

4) 9.83406

5) 3.39106

Exercise 3

Compute the center of curvature for $C(t) = \left\{-3t^2,\ 2 + 3t,\ 2t^2\right\}$ at the point $t = -2$.

1) {18.9615, 238.667, −59.3077}

2) {−65.0385, 28.6667, −87.3077}

3) {−37.0385, 140.667, 24.6923}

4) {−93.0385, 14.6667, −45.3077}

5) {32.9615, 168.667, 80.6923}

Exercise 4

Consider the vector field $F(x,y,z) = (0, z, -y)$ and the parameterized curve

$\sigma : [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (t, 2t, 2 + t)$

Compute $\int_\sigma F$.

1) 13.6 2) 32. 3) −11.2 4) 8.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 86

Exercise 1

Given the function

$f(x,y) = 3x^3 - 5y^3$ defined over the domain $D \equiv 27x^2 + 45y^2 \leq 2592$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.8****

3) The value of the minimum is ****.7****

4) The value of the minimum is ****.4****

5) The value of the minimum is ****.6****

Exercise 2

Compute $\int_D (2x + 2y)\,dx\,dy$ for $D = \{0 \leq 2y \leq 1,\ 0 \leq 9x + 6y \leq 7\}$

1) -0.932716

2) -0.432716

3) -1.13272

4) 0.667284

5) 0.367284

Exercise 3

Compute the mean curvature for $X(u,v) = \{2u, 2, v\}$ at the point $(u,v) = (5, 5)$.

1) $H(5, 5) = 1.81554$

2) $H(5, 5) = -4.80809$

3) $H(5, 5) = 0$

4) $H(5, 5) = 1.73075$

5) $H(5, 5) = 0.591301$

Exercise 4

Consider the vector field $F(x,y,z) = (0, -y, z)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v + 2v^2, 4v, u^2 + v)$

Compute $\int_X F.$

1) 22.6667 2) 26.6667 3) -23.3333 4) -9.33333

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 87

Exercise 1

Given the function

$f(x,y) = 5x^3 + y^3$ defined over the domain $D \equiv 15x^2 + 6y^2 \leq 156$, compute its absolute maxima and minima.

1) The value of the minimum is ****.6****

2) The value of the minimum is ****.5****

3) The value of the minimum is ****.0****

4) The value of the minimum is ****.7****

5) The value of the minimum is ****.4****

Exercise 2

Compute $\int_D (4y)\,dx\,dy$ for $D = \{0 \leq 7x - 8y \leq 1,\ 0 \leq 4x - y \leq 3\}$

1) -1.2368

2) 1.6632

3) 0.4632

4) 0.1632

5) 1.3632

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (1,4)$.

1) $H(1,4) = 7.42942$

2) $H(1,4) = -0.191752$

3) $H(1,4) = 3.8571$

4) $H(1,4) = 0.5$

5) $H(1,4) = -7.5595$

Exercise 4

Consider the vector field $F(x,y,z) = (-z, 0, 0)$ and the parameterized surface

$X: [0,1] \times [-1,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1, 2 + 2u^2, uv)$

Compute $\int_X F.$

1) 3.3 2) 0.1 3) -2.7 4) 0.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 88

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3 y}{(x^2+y^2)^{3/2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 6

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 1

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 49,\ z\ge 7\sqrt{x^2+y^2}\right\}$

1) 15.8841

2) 7.22006

3) 14.4401

4) −2.88802

5) 9.38608

Exercise 3

Compute the center of curvature for $C(t)=\left\{-3t^2,\ 1-2t,\ 2t^2\right\}$ at the point $t=-2$.

1) {67.0385, −41.4, 189.908}

2) {−139.962, −310.5, −58.4923}

3) {−57.1615, −310.5, 86.4077}

4) {−36.4615, −207., 24.3077}

5) {−139.962, −269.1, 65.7077}

Exercise 4

Consider the vector field $F(x,y,z)=(0,z,0)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2,2+t,2t)$

Compute $\int_\sigma F$.

1) −0.7 2) −1.7 3) 0. 4) −1.

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 89

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^3 - 3y^3}{2x + 2x^2 + x^3 - 2x^4 - 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -4

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -2

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \le 100,\ z \ge 7\ (x^2 + y^2)\}$

1) 2.22804

2) 20.0524

3) 22.2804

4) 6.68413

5) -20.0524

Exercise 3

Compute the center of curvature for $C(t) = \{-t, -t^2, t^2\}$ at the point $t = 2$.

1) {121.6, 38.95, 5.85}

2) {64., -12.25, 12.25}

3) {38.4, 45.35, -45.35}

4) {70.4, 26.15, 69.85}

5) {19.2, -57.05, 37.85}

Exercise 4

Consider the vector field $F(x,y,z) = (-x, 0, 0)$ and the parameterized curve

$\sigma: [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2 + 2t^2, 2, 1 + 2t)$

Compute $\int_\sigma F$.

1) -3.4 2) -0.8 3) 0. 4) -0.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 90

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^4+y^4}{-4x-2x^2+x^4-4x^5+2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 13

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 25

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 144,\ z\ge 13\left(x^2+y^2\right)\right\}$

1) 45.0942

2) 17.3439

3) 45.0942

4) 6.93758

5) 27.7503

Exercise 3

Compute the center of curvature for $C(t)=\left\{3t^2,\ -3+2t,\ -3t^2\right\}$ at the point $t=2$.

1) {-138.267, -494.7, -240.033}

2) {36.3333, -291., -36.3333}

3) {-225.567, -261.9, 21.8667}

4) {94.5333, -552.9, -298.233}

5) {94.5333, -436.5, 80.0667}

Exercise 4

Consider the vector field $F(x,y,z)=(0,-x,y)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(t+t^2,2t^2,2)$

Compute $\int_\sigma F$.

1) -2.66667 2) -11.3667 3) 3.63333 4) -2.96667

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 91

Exercise 1

Given the function

$f(x,y) = 5x^3 + y^3$ defined over the domain $D \equiv 30x^2 + 9y^2 \leq 804$, compute its absolute maxima and minima.

1) The value of the maximum is ****.3****

2) The value of the maximum is ****.1****

3) The value of the maximum is ****.0****

4) The value of the maximum is ****.9****

5) The value of the maximum is ****.6****

Exercise 2

Compute $\int_D (4x)\,dx\,dy$ for $D = \{0 \leq -8x + y \leq 4,\ 0 \leq 4x - 9y \leq 9\}$

1) 0.999308

2) 0.299308

3) −0.700692

4) −2.50069

5) 0.599308

Exercise 3

Compute the mean curvature for $X(u,v) = \{e^v \text{Cos}[u],\ e^v \text{Sin}[u],\ v\}$ at the point $(u,v) = (1,4)$.

1) $H(1,4) = -4.60427$

2) $H(1,4) = -4.39642$

3) $H(1,4) = 0.50629$

4) $H(1,4) = 5.62393 \times 10^{-8}$

5) $H(1,4) = 3.94356$

Exercise 4

Consider the vector field $F(x,y,z) = (x, -y, 0)$ and the parameterized surface

$X: [0,1] \times [-1,1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (2v, 2v^2, u)$

Compute $\int_X F.$

1) −8. 2) −36.8 3) −10.4 4) 14.4

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 92

Exercise 1

Given the function

$f(x,y) = 2x^3 - y^3$ defined over the domain $D \equiv 15x^2 + 6y^2 \leq 471$, compute its absolute maxima and minima.

1) The value of the maximum is ****.2****

2) The value of the maximum is ****.5****

3) The value of the maximum is ****.8****

4) The value of the maximum is ****.6****

5) The value of the maximum is ****.9****

Exercise 2

Compute $\int_D (4x)\,dx\,dy$ for $D = \{0 \leq -x - 4y \leq 3,\ 0 \leq 8x + 8y \leq 5\}$

1) 2.29167

2) -1.375

3) 3.4375

4) 4.8125

5) 0.

Exercise 3

Compute the mean curvature for $X(u,v) = \{2u, 2, v\}$ at the point $(u,v) = (9, 10)$.

1) $H(9, 10) = 2.4666$

2) $H(9, 10) = -5.37635$

3) $H(9, 10) = -3.78774$

4) $H(9, 10) = 0$

5) $H(9, 10) = -7.60187$

Exercise 4

Consider the vector field $F(x,y,z) = (x, 0, 0)$ and the parameterized surface

$X: [0, 1] \times [0, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1 + 2v, u^2, 3v^2)$

Compute $\int_X F.$

1) 32.9 2) 7. 3) 16.1 4) 5.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 93

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x+2y}{\sqrt{x^2+y^2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -48

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -2

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 121,\ z\geq 13\sqrt{x^2+y^2}\right\}$

1) 7.38993

2) 0.

3) 8.21103

4) 7.38993

5) 12.3166

Exercise 3

Compute the center of curvature for $C(t)=\left\{t^2,\ 2t^2,\ 3+t\right\}$ at the point $t=-2$.

1) {12.1, 24.2, 163.}

2) {−85.7, −122.5, 244.5}

3) {28.4, −73.6, 228.2}

4) {−4.2, −89.9, 32.6}

5) {44.7, 73.1, 81.5}

Exercise 4

Consider the vector field $F(x,y,z)=(-z,-y,0)$ and the parameterized curve

$\sigma:[-1,0]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(3t,1,1)$

Compute $\int_\sigma F$.

1) 0.3 2) 3.6 3) −3. 4) −0.3

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 94

Exercise 1

Given the function

$f(x,y) = -x^3 + y^3$ defined over the domain $D \equiv 9x^2 + 3y^2 \leq 336$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.8****

3) The value of the minimum is ****.0****

4) The value of the minimum is ****.2****

5) The value of the minimum is ****.1****

Exercise 2

Compute $\int_D (x + 3y)\,dx\,dy$ for $D = \{0 \leq 7x - 4y \leq 6,\ 0 \leq 7x - 2y \leq 9\}$

1) 13.1832

2) 23.9694

3) 0.

4) 7.19082

5) 11.9847

Exercise 3

Compute the mean curvature for $X(u,v) = \{3u, 3, v\}$ at the point $(u,v) = (2, 7)$.

1) $H(2, 7) = 7.3468$

2) $H(2, 7) = 7.1517$

3) $H(2, 7) = 5.21696$

4) $H(2, 7) = 0$

5) $H(2, 7) = -2.91303$

Exercise 4

Consider the vector field $F(x,y,z) = (x, 0, y)$ and the parameterized surface

$X: [-1, 0] \times [-1, 1] \longrightarrow \mathbb{R}^3$

$X(u,v) = (1 + 2u, 2v + v^2, 2 + 2u)$

Compute $\int_X F.$

1) 8. 2) -4. 3) 21.6 4) -12.8

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 95

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^4 - y^4}{3x + 6x^2 + x^4 + 6x^5 - 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 1

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to −8

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \le 225,\ z \ge 6\,(x^2 + y^2)\}$

1) −17.5736

2) 23.4315

3) 152.305

4) 58.5788

5) 105.442

Exercise 3

Compute the center of curvature for $C(t) = \{2t^2,\ 3 - 3t,\ -3t^2\}$ at the point $t = 3$.

1) {54.6923, 471., −82.0385}

2) {−133.708, 847.8, −317.538}

3) {195.992, 659.4, 153.462}

4) {478.592, 329.7, −129.138}

5) {7.59231, 800.7, −505.938}

Exercise 4

Consider the vector field $F(x,y,z) = (y, 0, 0)$ and the parameterized curve

$\sigma : [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (2 + t^2, 2t^2, 3)$

Compute $\int_\sigma F$.

1) 0. 2) 1.3 3) 3.2 4) −0.2

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 96

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2x^4+3y^4}{-9x-3x^2+x^4-3x^5+3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 241

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 257

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 25,\ z\ge 14\left(x^2+y^2\right)\right\}$

1) -0.557011

2) 6.40562

3) 3.06356

4) 2.78505

5) 6.12712

Exercise 3

Compute the center of curvature for $C(t)=\left\{-2t^2,\ t^2,\ 2t\right\}$ at the point $t=0$.

1) {-0.8, 0.4, 0}

2) {0, 0.9, -0.5}

3) {-1.3, -0.4, 0.5}

4) {-0.1, 0.2, -0.9}

5) {-0.7, 0.2, -0.9}

Exercise 4

Consider the vector field $F(x,y,z)=(z,0,y)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(t^2,4t^2,2t+2t^2)$

Compute $\int_\sigma F$.

1) -16. 2) -7.2 3) 8. 4) 33.6

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - hand for serial number: 97

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^4+3y^4}{6x+6x^2+x^4+6x^5-3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 50

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 110

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 81,\ z\ge 2\left(x^2+y^2\right)\right\}$

1) 160.894

2) 105.2

3) 160.894

4) 61.8825

5) -6.18825

Exercise 3

Compute the center of curvature for $C(t)=\left\{-2t,\ -3t^2,\ 3t^2\right\}$ at the point $t=3$.

1) {291.6, -858.933, 761.733}

2) {972., -81.3333, 81.3333}

3) {874.8, -178.533, -307.467}

4) {388.8, 404.667, 761.733}

5) {1458., -761.733, 858.933}

Exercise 4

Consider the vector field $F(x,y,z)=(0,y,-z)$ and the parameterized curve

$\sigma:[-1,1]\longrightarrow\mathbb{R}^3$

$\sigma(t)=(2,t+2t^2,3t)$

Compute $\int_\sigma F$.

1) 13.2 2) 15.6 3) 4. 4) 6.4

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Exam January-Call - hand for serial number: 98

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3 - y^3}{4x + 6x^2 + x^3 + 2x^4 - 2x^5 - 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -12

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -6

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \{x^2 + y^2 + z^2 \le 49,\ z \ge 5\ (x^2 + y^2)\}$

1) 7.58799

2) 18.2112

3) -1.77636×10^{-15}

4) 15.176

5) 16.6936

Exercise 3

Compute the center of curvature for $C(t) = \{-t^2, -2t^2, 3 - 3t\}$ at the point $t = 1$.

1) {−1.2, −14.1, 14.1667}

2) {3.3, −9.6, 15.9667}

3) {−7.5, −12.3, 6.96667}

4) {−3.9, −7.8, 9.66667}

5) {−5.7, −13.2, 5.16667}

Exercise 4

Consider the vector field $F(x,y,z) = (-x, y, 0)$ and the parameterized curve

$\sigma : [-1, 1] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (t^2, t^2, 3t^2)$

Compute $\int_\sigma F$.

1) −0.7 2) −2.6 3) 0. 4) −1.2

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Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^2 + 2xy + 2y^2}{\sqrt{x^2 + y^2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to -3

3) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit equal to 17

4) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \le 4,\ z \ge \sqrt{x^2 + y^2}\right\}$

1) 4.90747

2) 9.81495

3) 12.7594

4) 5.88897

5) 12.2687

Exercise 3

Compute the center of curvature for $C(t) = \left\{-t^2,\ 2 - t,\ t^2\right\}$ at the point $t = 1$.

1) {−4.25, 1., 8.25}

2) {−11.25, 18., 4.25}

3) {−1.25, 5., −5.75}

4) {−0.25, 5., −2.75}

5) {−3.25, 10., 3.25}

Exercise 4

Consider the vector field $F(x,y,z) = (-y, 0, -z)$ and the parameterized curve

$\sigma : [-1, 0] \longrightarrow \mathbb{R}^3$

$\sigma(t) = (1 + 2t^2, 2, 4t)$

Compute $\int_\sigma F$.

1) 12. 2) 15.6 3) 42. 4) 37.2

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Exercise 1

Given the function

$f(x,y) = 3x^3 + 3y^3$ defined over the domain $D \equiv 18x^2 + 9y^2 \leq 324$, compute its absolute maxima and minima.

1) The value of the maximum is ****.0****

2) The value of the maximum is ****.3****

3) The value of the maximum is ****.9****

4) The value of the maximum is ****.5****

5) The value of the maximum is ****.2****

Exercise 2

Compute $\int_D (2x)\,dx\,dy$ for $D = \{0 \leq -5y \leq 5,\ 0 \leq -6x + 8y \leq 3\}$

1) 0.383333

2) 0.983333

3) -0.516667

4) -0.916667

5) -2.81667

Exercise 3

Compute the mean curvature for $X(u,v) = \{e^v \cos[u], e^v \sin[u], v\}$ at the point $(u,v) = (4,5)$.

1) $H(4,5) = -3.73475$

2) $H(4,5) = 1.03051 \times 10^{-9}$

3) $H(4,5) = -6.63001$

4) $H(4,5) = -4.0472$

5) $H(4,5) = -4.32203$

Exercise 4

Consider the vector field $F(x,y,z) = (y, x, 0)$ and the parameterized surface

$X: [0,1] \times [-1,0] \longrightarrow \mathbb{R}^3$

$X(u,v) = (3uv, uv, 2u)$

Compute $\int_X F.$

1) -2.66667 2) -3.56667 3) -11.6667 4) -10.4667